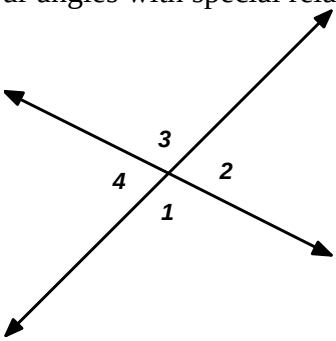
9.3 More About Angles

Two intersecting lines form four angles with special relationships:

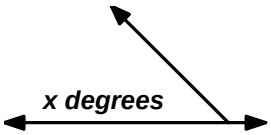


$\angle 1$ and $\angle 3$ are VERTICAL ANGLES. Their measures are equal.
$\angle 2$ and $\angle 4$ are VERTICAL ANGLES. Their measures are equal.

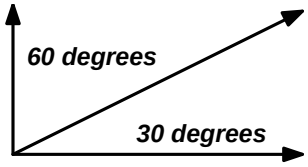
THEOREM 9-2: VERTICAL ANGLES ARE CONGRUENT.

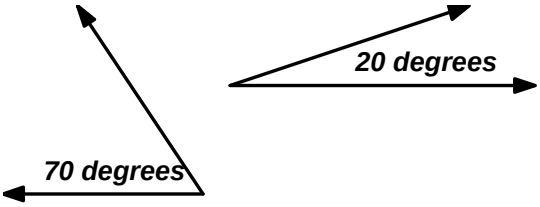
SUPPLEMENTARY ANGLES are two angles whose measures add up to 180°. Name some pairs of supplementary angles in the above figure. _____

In the figure below, what could you call the measure of the angle that is supplementary to the angle marked x degrees? _____

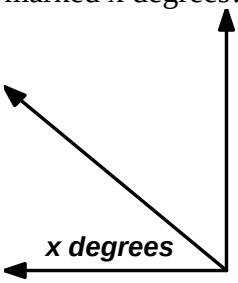


COMPLEMENTARY ANGLES are two angles whose measures add up to 90°.
Examples:



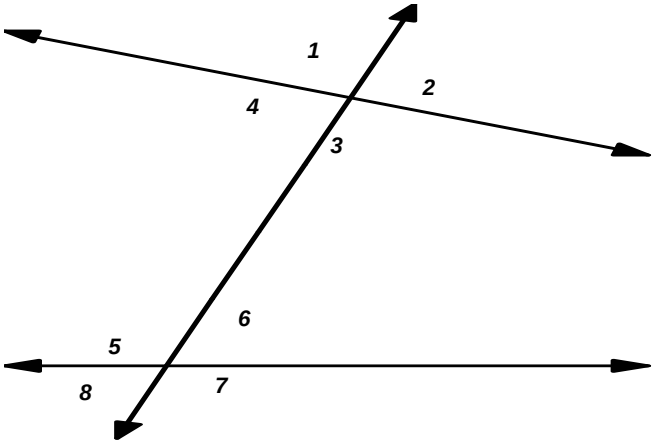


In the figure below, what could you call the measure of the angle that is complementary to the angle marked x degrees?



TRANSVERSALS

A TRANSVERSAL is any line that intersects a pair of lines.



Transversals form these types of angles:

INTERIOR ANGLES:

EXTERIOR ANGLES:

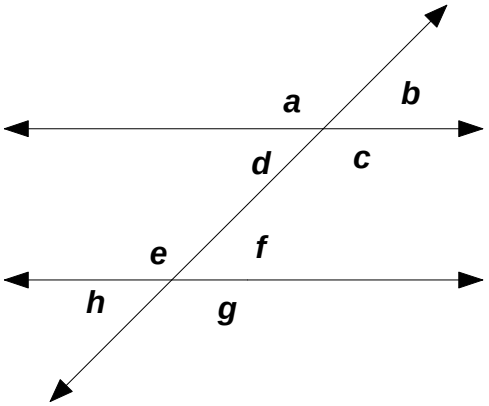
ALTERNATE INTERIOR ANGLES:

ALTERNATE EXTERIOR ANGLES:

CORRESPONDING ANGLES:

THEOREM 9-3: If any two distict coplanar lines are cut by a transversal, then a pair of **corresponding angles**, **alternate interior angles**, or **alternate exterior angles** are congruent if, anf only if, the lines are parallel.

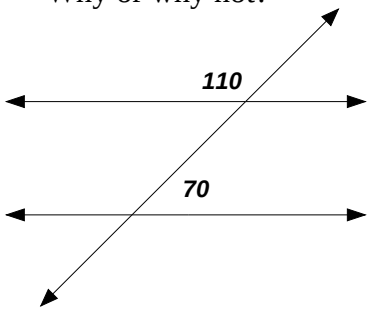
Given these two parallel lines cut by a transversal:



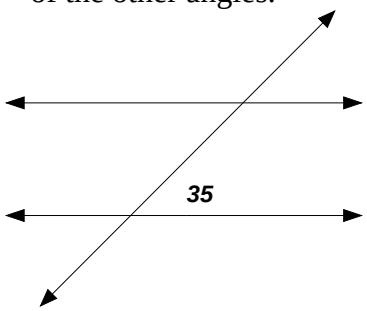
Name all pairs of congruent angles:

Two more problems:

1) Are these 2 lines parallel? Why or why not?



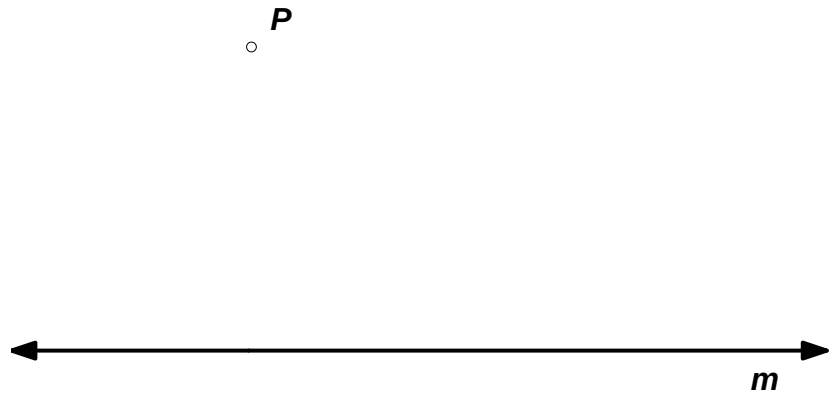
2) Given that these lines are parallel, find the measures of the other angles.



CONSTRUCTING PARALLEL LINES

(A method used by architects.)

Construct a line parallel to line m and passing through point P:



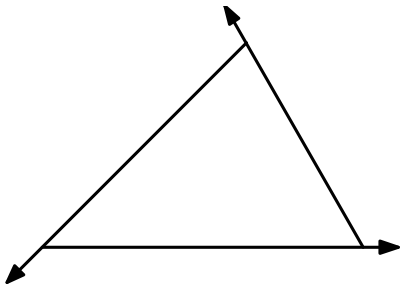
THE SUM OF THE MEASURES OF THE ANGLES OF A TRIANGLE

You probably already know that this sum is 180°.
In this section, we see three ways of showing that the sum of the three angles of any triangle is 180°.

First: See figure 9-23 on page 603, where the angles are actually torn out and placed along a straight line.

Second: Use what you know about parallel lines. See figure 9-24.

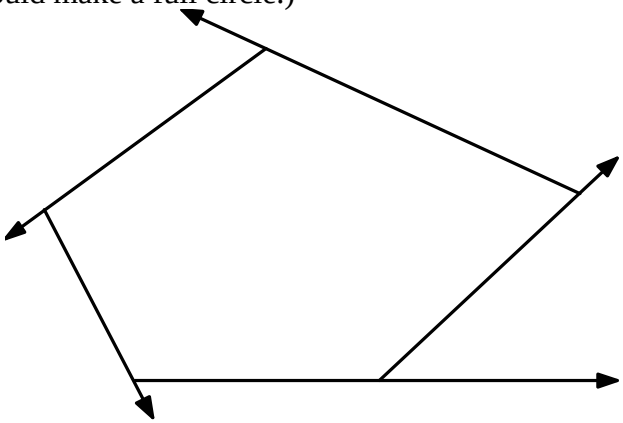
Third: The last method involves walking around a triangle and adding the number of degrees in our "turns". Altogether, we would have turned 360°. This means that the sum of the exterior angles of the triangle is 360°.



Now consider adding these exterior angles to the three interior angles. We would have 3 pairs of supplementary angles. So the total is $3(180) = 540°$. If you then subtract the total of the exterior angles from 540°, you have the sum of the interior angles. $540° - 360° = 180°$.

So, now we can formally state:
THEOREM 9-4 The sum of the measures of the interior angles of a triangle is 180°.

In general, for any convex polygon, if you walk around it, the sum of the turning angles is 360°. (You would make a full circle.)



This brings us to:
THEOREM 9-5: The sum of the measures of the exterior angles (one at each vertex) of a convex polygon is 360°.

A convex n-*gon* has n interior angles and n exterior angles (one per vertex). So what would be the sum of all of its interior and exterior angles?

Use this and the fact that the sum of its exterior angles is 360° to develop a formula for the sum of its interior angles.

THEOREM 9-6:
a) The sum of the measures of the interior angles of any convex polygon with n sides is:

b) The measure of a single interior angle of a regular n-*gon* is: