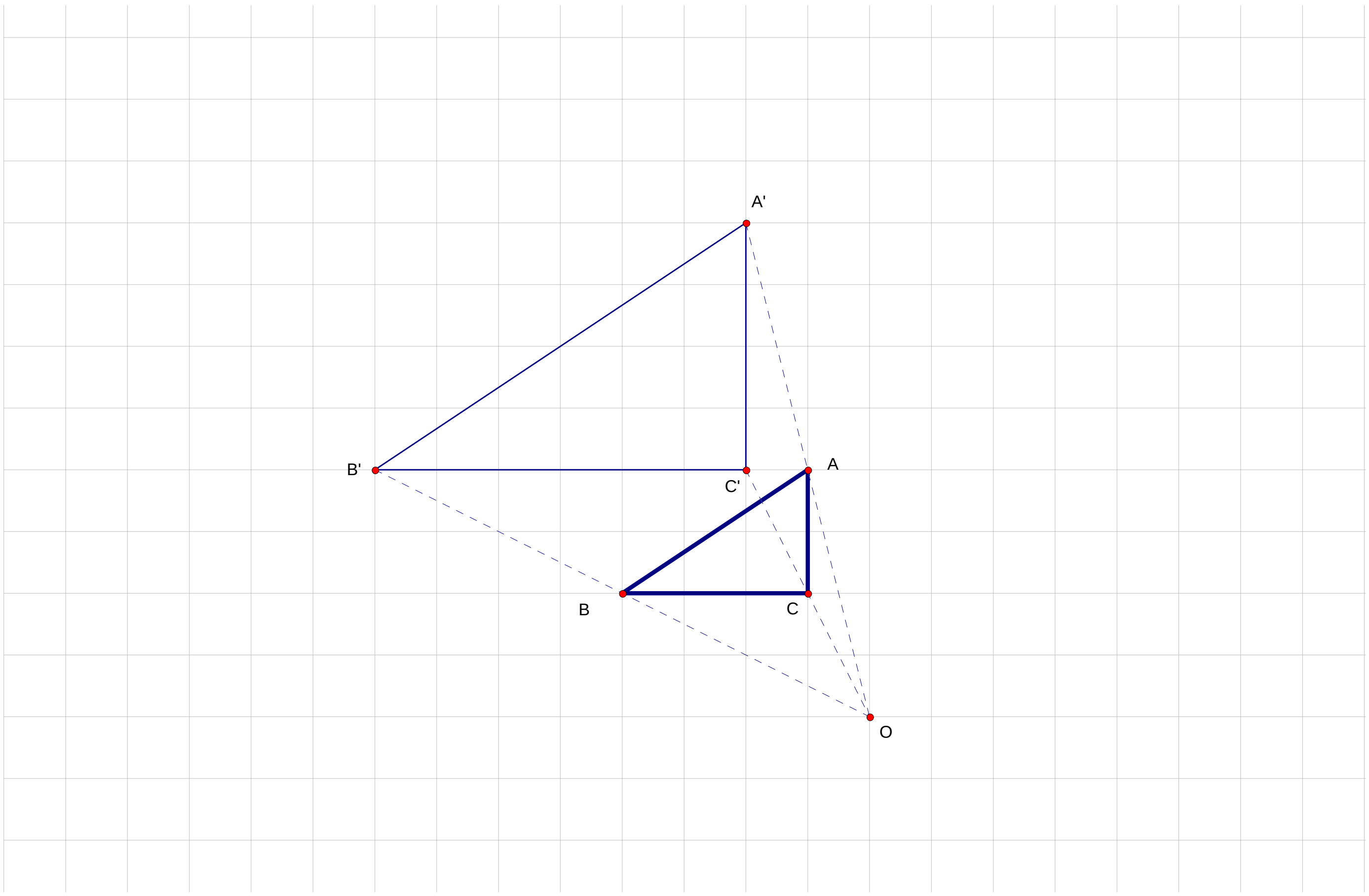
12.3 Size Transformations

So far, the geometric motions that we have studied have been isometries. They all produced images that were the same size and shape. An example of a transformation that does not preserve size is that of projecting a slide onto a screen. Shape is preserved, but not size.

In the figure below, △ABC has been transformed into △A'B'C'. How do the lengths of the corresponding sides of the two triangles compare?



Point O is called the **center** of this size transformation and the **scale factor** is _______.
Note: Points O, A, and A' are collinear and $OA' = 2 \cdot OA$.
Are these two triangles congruent?_________ How could you describe them?_________________________

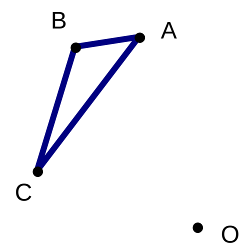
DEFINITION OF A SIZE TRANSFORMATION: A **size transformation** from the plane to the plane with **center O** and **scale factor r** ($r > 0$) is a transformation that assigns to each point A in the plane a point A', such that O, A and A' are collinear and $OA' = r \cdot OA$ and so that O is not between A and A'.

Now to project the image of a point under such a transformation involves two steps. We will try an example. Project the image of point A with center O and scale factor 3:

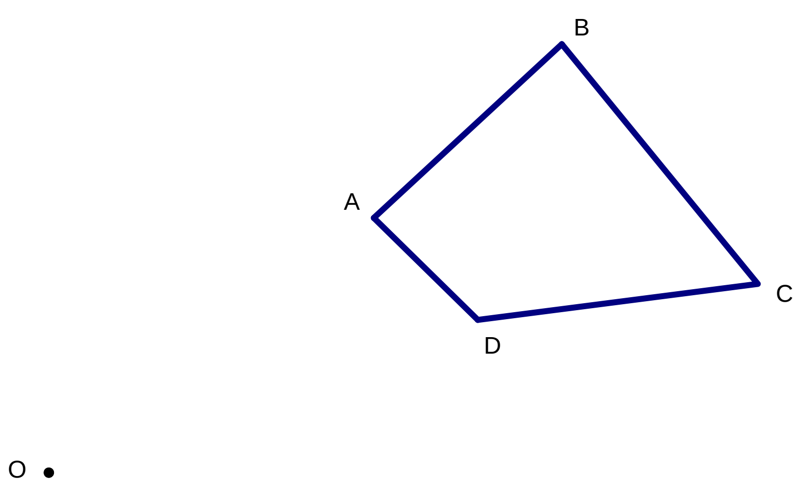
Two Steps:

1) Construct $\overrightarrow{OA}$.
2) Consider OA to be one unit, and mark off two more "units" on $\overrightarrow{OA}$.
   Thus OA ' = 3OA, and A' is the image of A.

To transform a polygon the same way, you would transform each vertex and join them with line segments. Transform △ABC with center O and scale factor 2:



Now try to transform quadrilateral ABCD with center O and scale factor 1/2:



What do you notice about the corresponding sides of the two quadrilaterals?______________________________

What do you notice about the corresponding angles?________________________________

THEOREM 12-1: A size transformation with center O and scale factor r ($r > 0$), has the following properties:

1. The image of a line segment is a line segment parallel to the original segment and r times as long.
2. The image of an angle is an angle congruent to the original angle.

So the image of a polygon under a size transformation is always going to be a <u>similar</u> polygon. Why??

Let's look at two examples in the textbook on page 861. Examples 12-9 and 12-10. In the first example, we are asked to show that $\triangle$ABC is the image of $\triangle$ADE under a size transformation. In the second example, we are asked to show that $\triangle$ABC is the image of $\triangle$APQ under a succession of isometries with a size transformation.

We now can use a different definition of similar figures:

DEFINITION OF SIMILAR FIGURES: Two figures are similar if it is possible to transform one onto the other by a sequence of isometries followed by a size transformation.