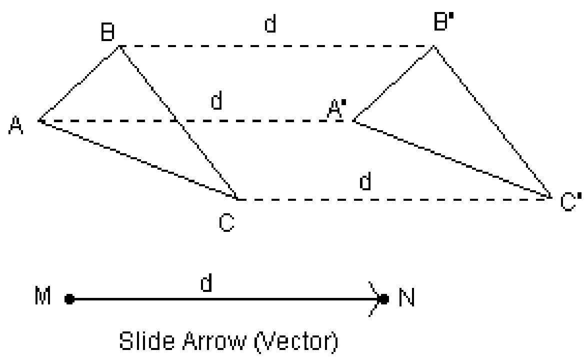
12.1 Translations and Rotations

There are two types of motions, or transformations, in this section - **translations** (slides) and **rotations** (turns).

A simple example of a translation can be found on page 830, figure 12-1. A child facing front slides down a slide, without twisting or turning. In the figure the blue line shows the direction of his slide. This line is the slide line; it is marked with a slide arrow (or vector). In this type of motion, the original figure is a set distance away from the image.

A TRANSLATION is a motion of a plane that moves every point of the plane a specified distance in a specified direction along a straight line.

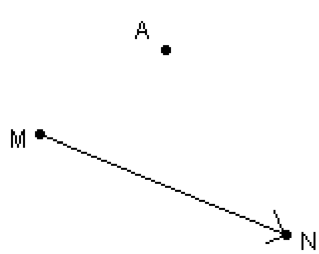
Here is another example:



A translation will preserve the size and shape of the figure involved. A transformation that does this is called an ISOMETRY, or rigid motion.

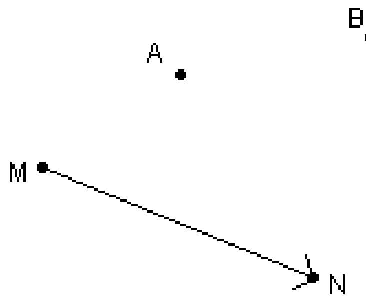
You can construct the image of a figure under a translation using tracing paper or a compass and straightedge. We will explore the compass and straightedge method.

- First, a translation of a single point:

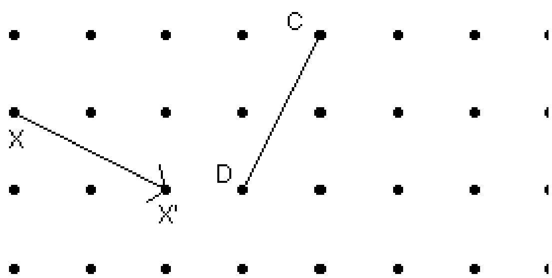


The image of A will be A', so $\overline{AA'} \parallel \overline{MN}$ and $\overline{AA'} \cong$ MN.
So, basically, we need to construct a parallelogram with vertices A, A', M and N.
Here are the steps: 1) Measure $\overline{MA}$.
2) Use this measure to mark an arc from point N.
3) Measure $\overline{MN}$.
4) Use this measure to mark an arc from point A.
5) The intersection of the two arcs is the image of A, or A'.

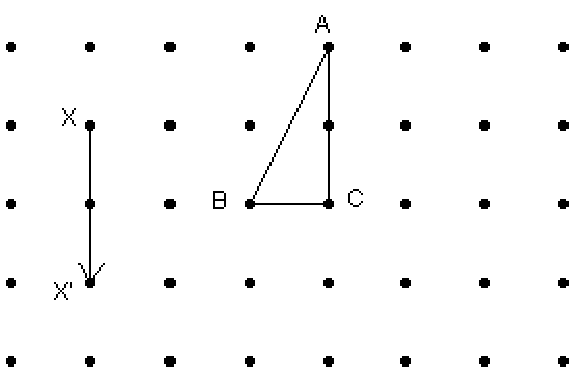
• Second, explain how you would translate $\overline{AB}$ using the same method:



Grid paper gives us a good tool for finding images.
Example 1: Find the image of $\overline{CD}$ under the translation from X to X'



Example 2: Find the image of △ABC under the translation from X to X'



We can also have translations defined in the coordinate plane. See figure 12-5 on page 833. Under that translation, each point of △ABC has been moved to the right 5 units and down 2 units.

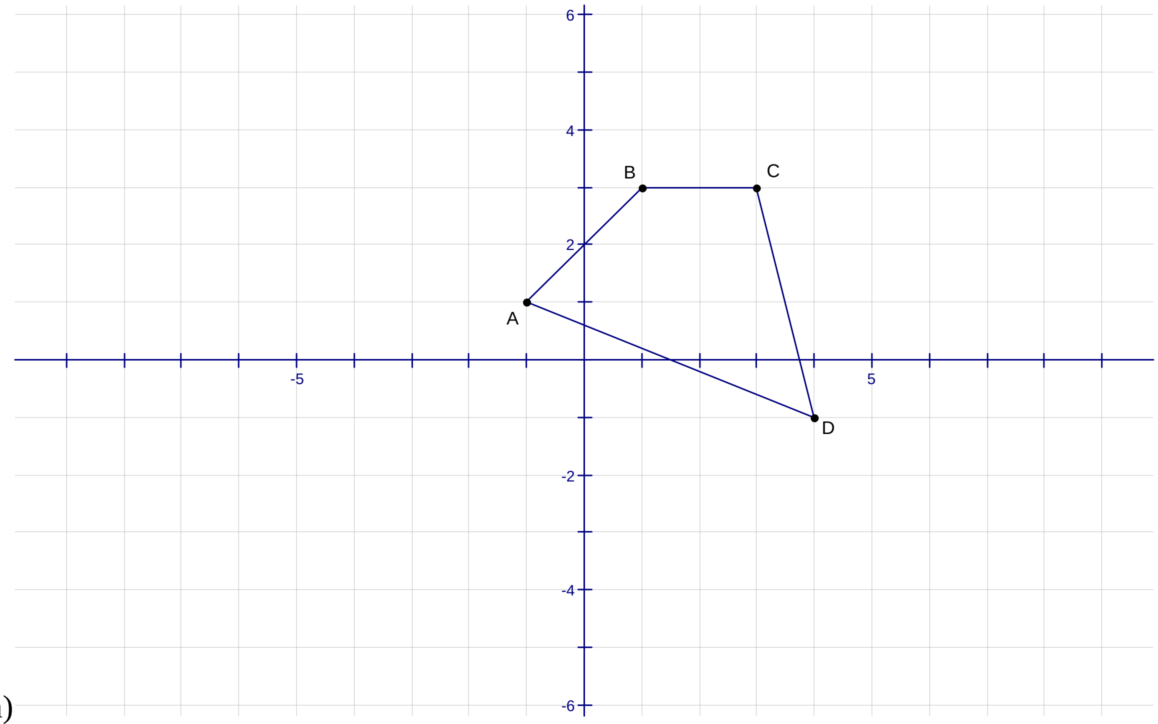
<u>Definition of a Translation in a Coordinate System</u>: A translation is a function from the plane to the plane such that to every point (x, y) corresponds the point (x + a, x + b) for real numbers a and b.

Symbolized **(x, y)→(x + *a*, y + *b*)**. This means that the point (x + a, y + b) is the image of the point (x, y).

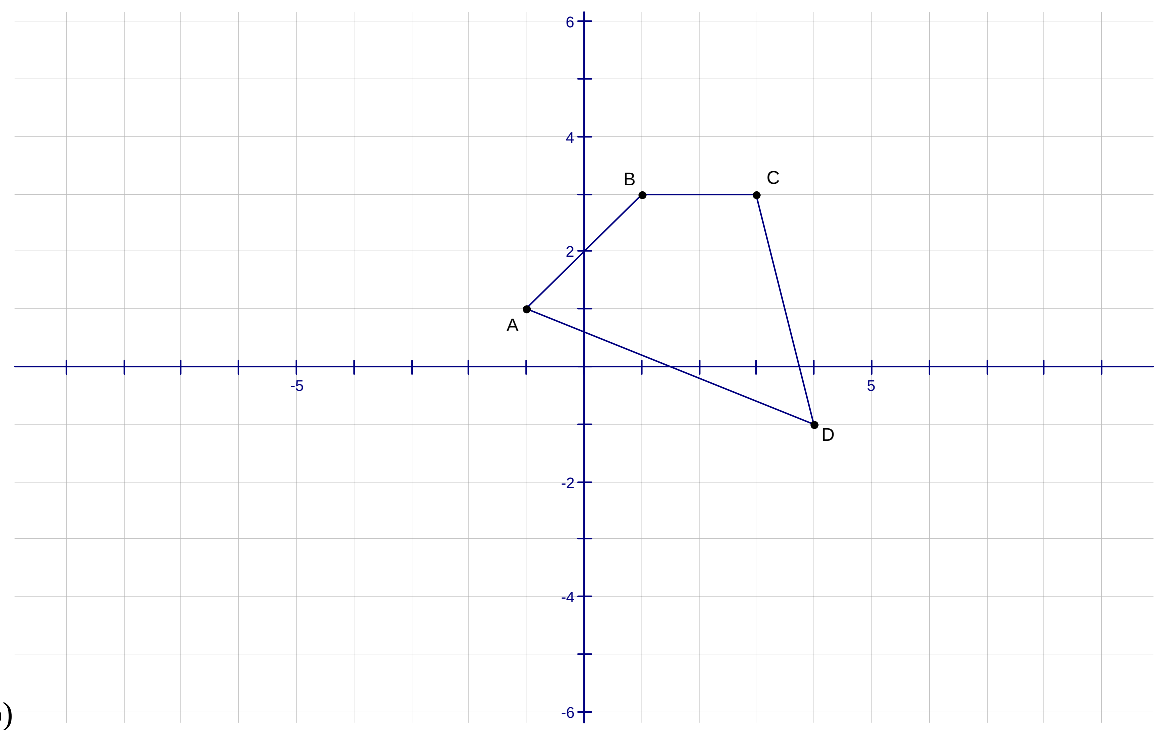
Example: Find the image of quadrilateral ABCD under each of these translations:

a) $(x, y) \rightarrow (x + 2, y - 4)$

b) A translation determined by the slide arrow from X (0, 0) to X' (− 3, 2)



a)



b)

A ROTATION , or turn, is also an isometry. Figure 12-8 on page 834 shows a rotation of a figure (the letter F) about a point O. The figure has been rotated counterclockwise 30°. The point O is called the **turn center**, and the angle is called the **turn angle**.

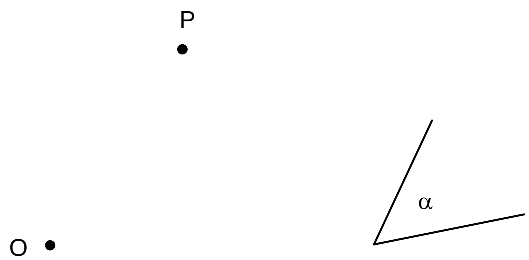
Tracing paper can be used to construct a rotation. We could trace the figure, then hold one point fixed and rotate (or turn) the paper. We will try an example.

Definition of a Rotation: A rotation is a motion of a plane determined by holding one point (the center) fixed and rotating the plane about this point by a certain amount in a certain direction.

To determine a rotation, three things must be known:

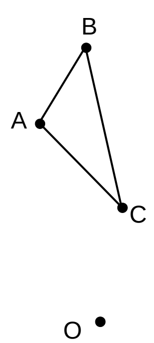
1) the turn center
2) the direction of the turn (clockwise or counterclockwise)
3) the amount of the turn

Now let's try constructing some rotations using compass and straightedge.
First: Rotate point P counterclockwise α degrees about turn center O:

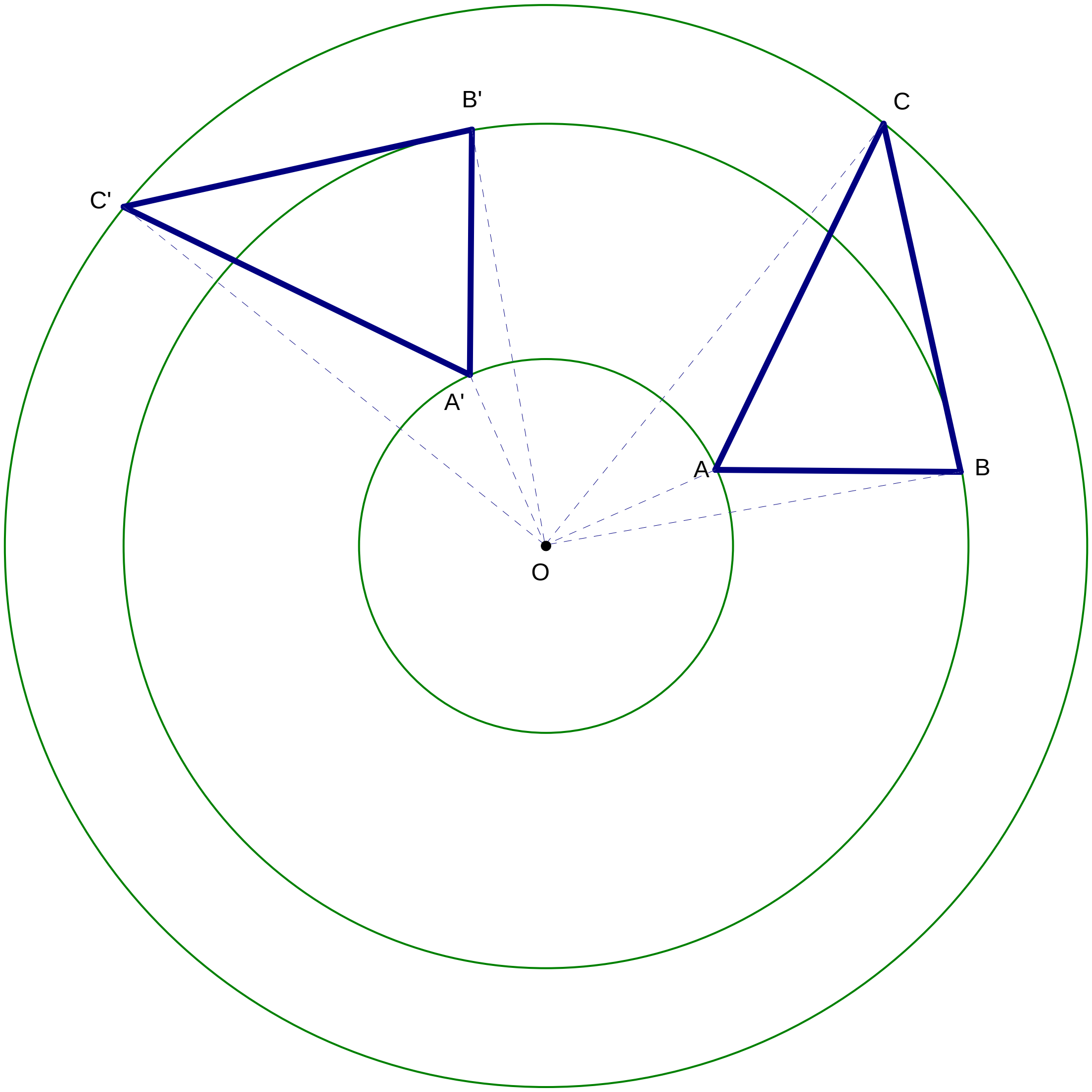


Points that are rotated move along a circle. So, what we need here is to construct a circle centered at point O and having radius $\overline{OP}$. Then the second step will be to copy the angle onto $\overline{OP}$, with its vertex at O.

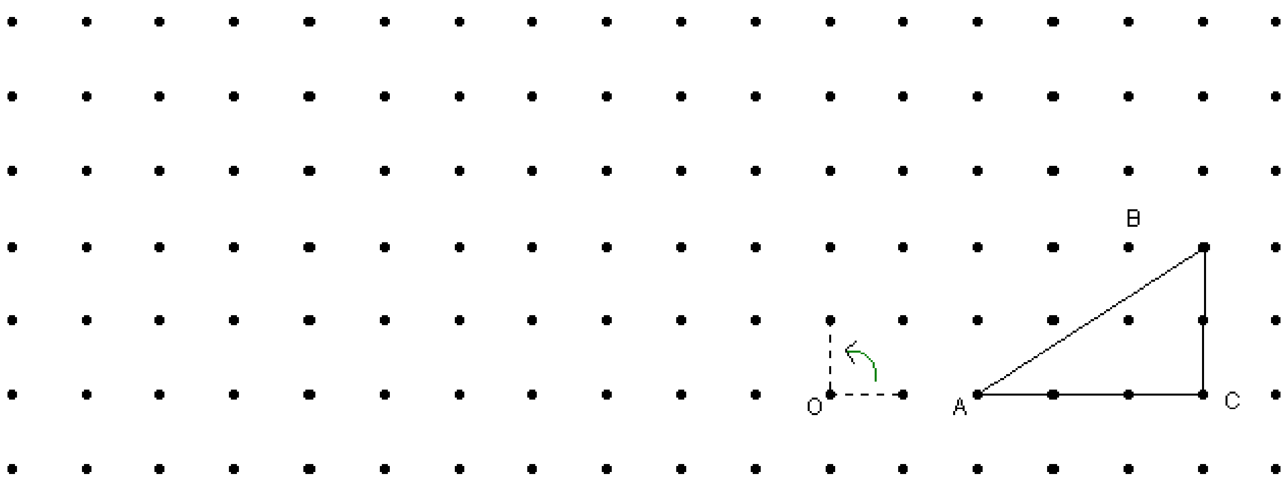
Second: Rotate $\triangle ABC$ clockwise α degrees (from above example) about turn center O:



What Happens When a Figure is Rotated About a Center O
(Note the circular paths of points A, B and C)



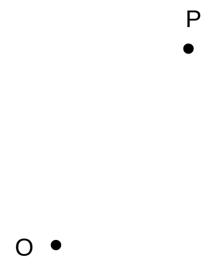
With certain angles of rotation, like 90°, dot paper can be used. Rotate this triangle counterclockwise 90° about center 0:



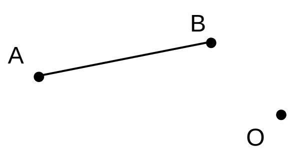
A 360° rotation in either direction would do what to a figure?

A 180° rotation is called a half-turn. Does the direction of rotation matter with the outcome of a half-turn?

Rotate point P a half-turn about center O:



Now rotate $\overline{AB}$ a half-turn about center O:



Look at figure 12-13 on page 837 for some examples of half-turns.