11.4 Surface Areas

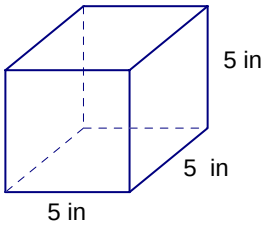
There are many real-life problems that involve finding areas – buying carpet, laying a hard-wood floor, painting or wallpapering a room, to name a few. A lot of area problems involve finding surface areas of three-dimensional figures such as cylinders, prisms, cones, spheres and pyramids. To find these surface areas, we work with areas of two-dimensional pieces of the 3-d figures. We will use what are called $nets$: 2-d patterns that can be used to construct 3-d figures.

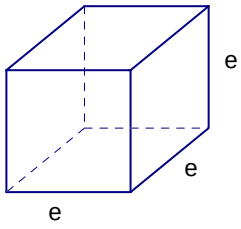
SURFACE AREA OF A RIGHT PRISM

A cereal box is a right prism. In fact, it is a right _____ prism. Consider the problem of figuring out how much cardboard is needed to construct it. One way to solve the problem is to cut the box in such a way as to be able to lie it down flat. This results in a net. We can then see that it is made up of what kinds of figures?_____ How many?_____ Are any of them the same size?_____

Let's look at a cereal box and its net.

The surface area of a cube can be calculated this way also. It has how many sides?_____ Are they all the same size?_____ What would be the surface area of a cube having an edge of 5 in? _____ What about a cube having an edge of size e?_____





In general, to calculate the surface area of a cube with edge e units, S.A. = _____

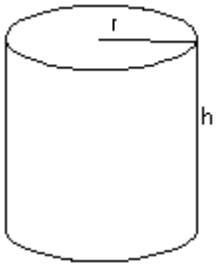
In general, finding the surface area of right prisms involves finding the lateral surface area and the sum of the areas of the bases. Adding these together results in the surface area.

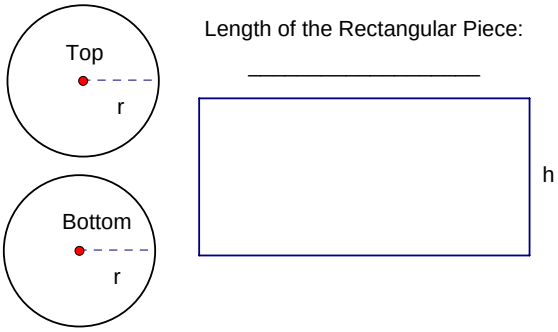
Look at figure 11-57 on page 787. The figure shows a right pentagonal prism with its net. Notice there are 5 rectangles (lateral sides) and two pentagonal bases.

SURFACE AREA OF A CYLINDER

An oatmeal box is a right circular cylinder. If you cut out the top and bottom, cut vertically through the remaining open cylinder, you have a net of the cylinder. What shapes do you have?

So to find the surface area of a cylinder requires finding the areas of these three figures. Let's consider a cylinder having radius measure r units and height h units. What is the formula for its surface area?



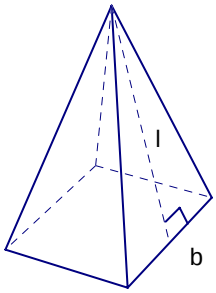


In general, the formula for finding the surface area of any right circular cylinder is:
S.A. $=$

Let's use our calculators to find the area of the oatmeal box. Its radius measures $1\frac{13}{16}$ in and its height measures $7\frac{1}{4}$ in.

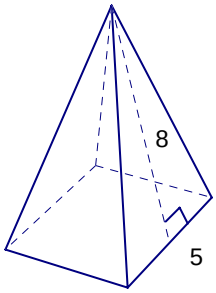
SURFACE AREA OF A PYRAMID

The surface area of a pyramid is the sum of the areas of its lateral faces and the area of its base. A right regular pyramid has lateral faces that are congruent triangles and a base that is a regular polygon. For example, here is a right square pyramid:



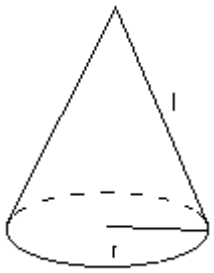
Note: Each triangle has altitude l, called the $slant\ height$. This altitude is different from the altitude of the pyramid. The sides of the base are all the same since it is a regular polygon. The area of each of the triangles is $\frac{1}{2}bl$. If the base is a regular polygon with n sides, the the lateral surface area would be $n \cdot \frac{1}{2}bl$. So the total surface area of a right regular pyramid is: S.A. $=$ B $+ n \cdot \frac{1}{2}bl$ where B is the area of the base.

Example: Find the surface area of the following right square pyramid:



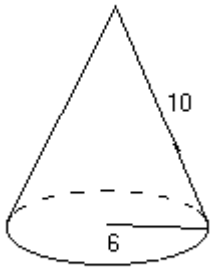
SURFACE AREA OF A CONE

Again the surface area of this figure involves adding the areas of its base and its lateral surface. The base is, of course, a circle, so the area of its base is πr^2, where r is the measure of the radius. The area of the lateral surface is πrl, where l is the $slant\ height$ of the cone.



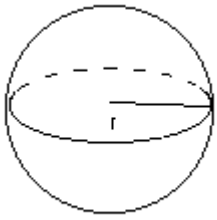
Again, note that the $slant\ height\ l$ is different from the height of the cone.

Example: Find the surface area of this right circular cone:



SURFACE AREA OF A SPHERE

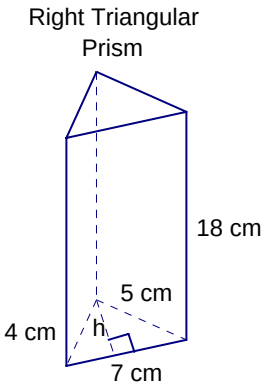
The surface area of a sphere is four times the area of a $great\ circle$ of the sphere. A great circle is any circle whose center and radius are the same as that of the sphere itself. So, the surface area of sphere with radius r is: S.A. $= 4\pi r^2$



Calculate the surface area of a sphere of radius 12 in.

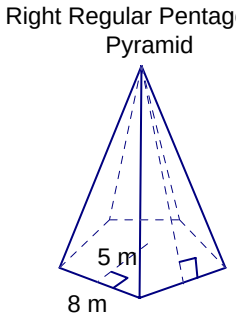
Find the surface area of each of these figures:

1.

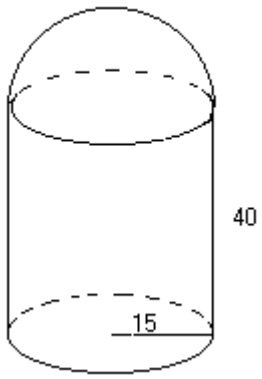


Altitude of the triangular base, h, equals 2.8 cm

2.



3.



4.