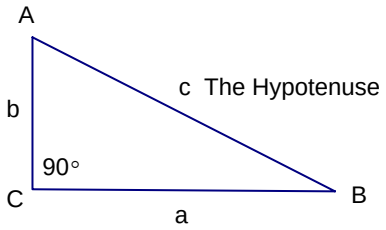
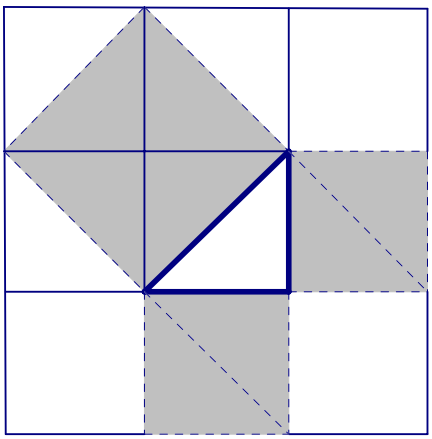
11.3 The Pythagorean Theorem

In a right triangle, the side opposite the right angle is called the hypotenuse. The other two sides are called the legs.



The Pythagorean Theorem states that the area of the square with the hypotenuse of a right triangle as its side is equal to the sum of the areas of the squares with the legs as sides. Here is one example:



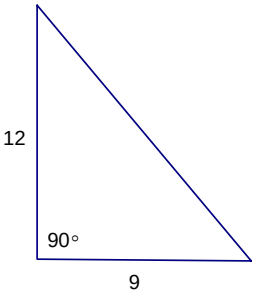
This is a special case where the right triangle is also isosceles. Can you see that the areas of the two smaller squares added together does equal the area of the larger square?

Theorem 11-1: The Pythagorean Theorem If a right triangle has legs of length a and b, and hypotenuse of length c, then $\boldsymbol{a^2 + b^2 = c^2}$.

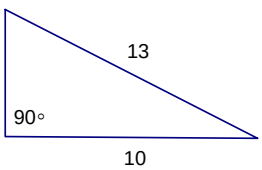
There are hundreds of proofs that have been done over the years. You can read about one proof on page 679.

Now a typical problem where the Pythagorean Theorem is used involves finding one side of a right triangle, given the measures of the other two sides.

Examples 1) Find the measures of the missing sides:



a)



b)

2) Find the measure of the diagonal of a rectangle whose length is 6 feet and width is 4 feet.

3) What is the measure of one side of a 35" square TV screen?

4) Try example 11-14 on page 774.

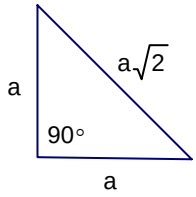
THERE ARE TWO SPECIAL TYPES OF RIGHT TRIANGLES:.

First, an Isosceles Right Triangle. Sketch one. What are the measures of the angles?

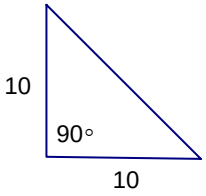
Let's consider an isosceles right triangle whose congruent sides are one unit long. What would be the measure of its hypotenuse?

Now consider the isosceles right triangle whose congruent sides measure a units. What is the measure of its hypotenuse?

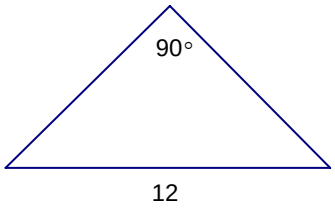
PROPERTY OF 45°-45°-90° TRIANGLE: If the length of each leg is a, then the length of the hypotenuse is $a\sqrt{2}$.



Examples: Find the measures of the missing sides:

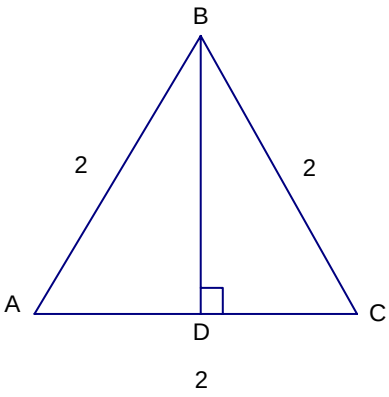


a)



b)

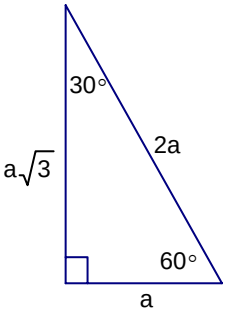
Second For the second special right triangle, we start with an equilateral triangle and draw the altitude to the base as shown:



What can you conclude about △BDC? What are the measures of its angles?

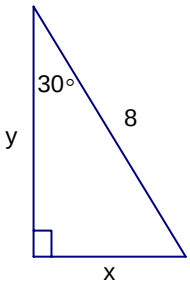
What is the measure of its hypotenuse?

PROPERTY OF 30°-60°-90° TRIANGLES The length of the hypotenuse is two times as long as the length of the side opposite the 30° angle, and the leg opposite the 60° angle is $\sqrt{3}$ times the shorter leg.

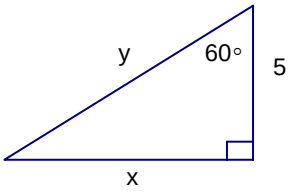


Examples:

1) Find x and y:



2) Find x and y:

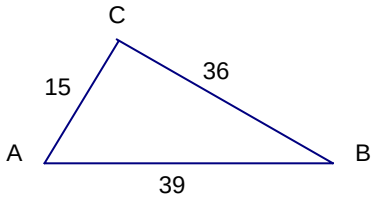


Now if the measures of the sides of a right triangle satisfy $a^2 + b^2 = c^2$, is the triangle a right triangle?

This is the Converse to the Pythagorean Theorem. (Theorem 11-2, page 684.)

Examples:

1) Is $\angle C$ a right angle?



2) Can 2, $\sqrt{5}$, and 3 be the lengths of the sides of a right triangle?