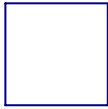


11.2 Areas of Polygons and Circles

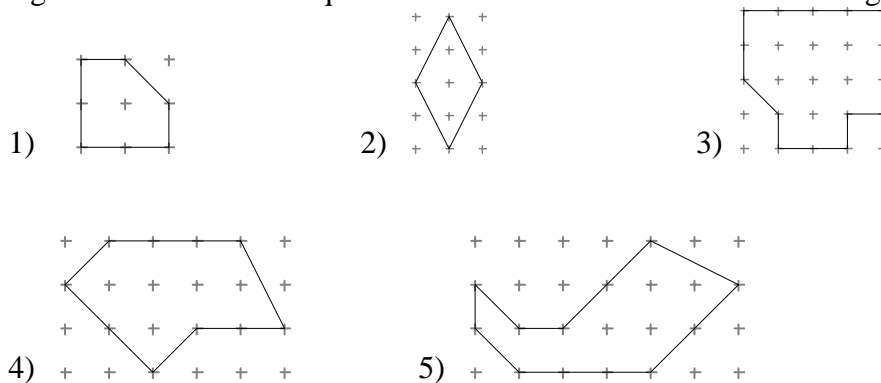
Area: Surface area of the interior of a figure. We use a square as the base measure and we see how many of these it takes to completely cover the region being measured.

One square inch is a square which measures one inch on each side. We write 1 in^2 for short.

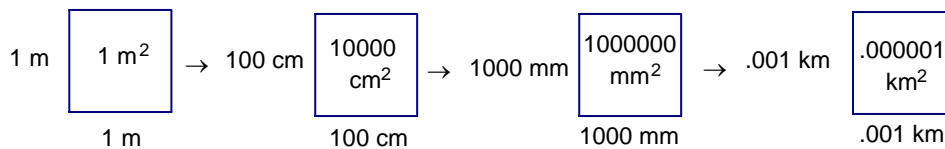


Areas on a Geoboard

Before you teach formulas to elementary students, it is good to have them work with geoboards. On a geoboard, the distance between 2 adjacent nails in a row or column is 1 unit. We can find areas of figures on a geoboard in terms of square units. Let's find the areas of these regions on dot paper:



Converting Units of Area One square meter is a square measuring a meter on each side. These are all equal to one square meter:



Here is a table relating the different metric units of measure to the square meter:

Unit	Symbol	Relationship to Square Meter
square kilometer	km^2	1,000,000 m^2
*square hectometer	hm^2	10,000 m^2
*square dekameter	dam^2	100 m^2
square meter	m^2	1 m^2
*square decimeter	dm^2	.01 m^2
square centimeter	cm^2	.0001 m^2
square millimeter	mm^2	.000001 m^2

* not often used

Try these conversions: $3 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$

$15,000,000 \text{ m}^2 = \underline{\hspace{2cm}} \text{ km}^2$

We can also do conversions among English units of measure:

Unit of Area	Equivalent of Other Units
1 ft ²	$\frac{1}{9}$ yd. ² or 144 in. ²
1 yd ²	9 ft ²
1 mi ²	3,097,600 yd ² or 27,878,400 ft ²

Now try these conversions: 3 yd² = _____ ft²

4,000,000 yd² = _____ mi²

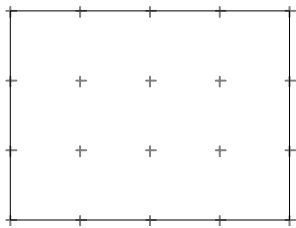
Land Measure In the English system, the most common measure is the acre, which is equivalent to 4840 yd². 640 acres is equivalent to one square mile.

In the metric system, a unit that is 10 m on a side is called an are, and denoted by a. A hectare is 100 a. For very large land measures, the square kilometer is used.

Unit of Area	Equivalent in Other Units
1 a	100 m ²
1 ha	100 a or 10,000 m ²
1 km ²	1,000,000 m ²
1 acre	4840 yd ²
1 mi ²	640 acres

What would be the area in acres of a square field that measures 600 yd on a side? _____

Area of a Rectangle After working with geoboards and measuring the areas of rectangles by counting the square units, the elementary student will readily see a quicker way of calculating the area of a rectangle.

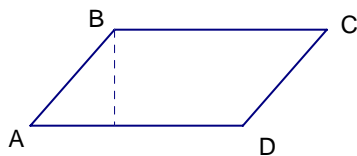


Area = _____

FORMULA FOR THE AREA OF A RECTANGLE WITH LENGTH l AND WIDTH w:

Example: Find the area of a rectangle with width 10 in and length 2 ft.

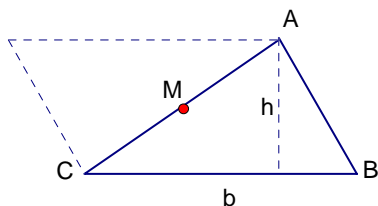
Area of a Parallelogram In parallelogram ABCD, construct a perpendicular from B to \overline{AD} . Label the point of intersection on \overline{AD} point E. Then slide $\triangle ABE$ to the right until point B coincides with point C. The resulting figure is a rectangle whose area is the same as the area of the parallelogram.



FORMULA FOR THE AREA OF A PARALLELOGRAM WITH BASE b AND HEIGHT h :

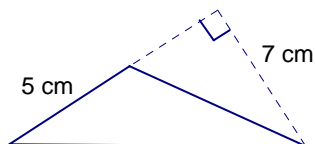
Example: Find the area of parallelogram ABCD above if the height is 4 cm and $AD = 10$ cm.

Area of a Triangle Consider $\triangle ABC$ with base b and height h . If we found M , the midpoint of \overline{AC} , and rotated $\triangle ABC$ 180° about M , we would get a parallelogram whose base is b and height is h . So what would be the area of the triangle?



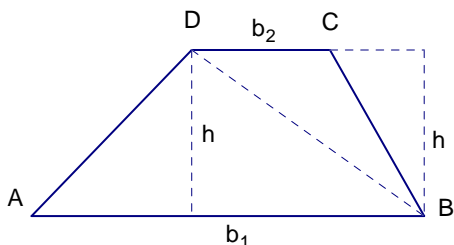
FORMULA FOR THE AREA OF A TRIANGLE WITH BASE b AND ALTITUDE h :

Example: Find the area of the given triangle:



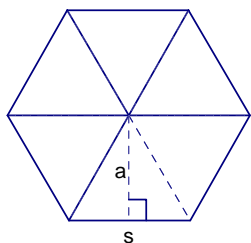
We can find the areas of some polygons by partitioning them into triangles.

Area of a Trapezoid Construct one of the diagonals of the trapezoid and then construct the altitudes to the 2 triangles formed, as in the diagram:



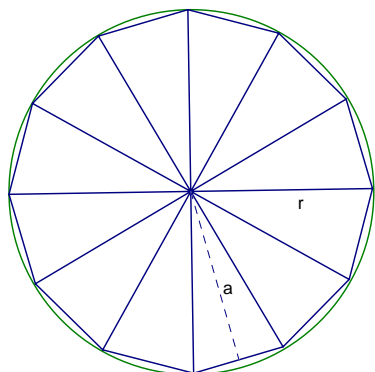
THE FORMULA FOR THE AREA OF A TRAPEZOID WITH BASES b_1 AND b_2 AND HEIGHT h IS:

Area of a Regular Polygon Partition the regular polygon into triangles. Consider this figure of a regular hexagon with side measure s . Note: the measure a (altitude of one of the triangles) is called the apothem.



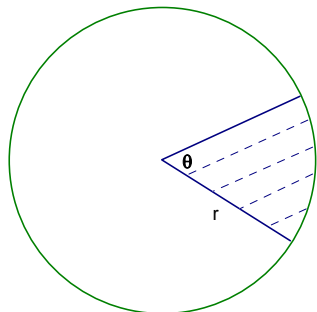
THE FORMULA FOR THE AREA OF A REGULAR POLYGON WITH APOTHEM a AND PERIMETER p IS:

Area of a Circle We can examine a related problem to find a formula for the area of a circle. Consider a regular polygon inscribed in a circle. As the number of sides increases, the polygonal area approximates the circular area. The perimeter p is close to circumference of the circle, $2\pi r$, and the apothem a is close to the radius r . So $A = \frac{1}{2}ap \approx \frac{1}{2} \cdot r \cdot 2\pi r = \pi r^2$



FORMULA FOR THE AREA OF A CIRCLE WITH RADIUS r IS:

Area of a Sector of a Circle A sector is a fractional part of a whole circle. What part depends on the central angle. If the central angle measures 120° , then the sector is _____ of the circle. If the measure of the central angle is 60° , then the sector is _____ of the circle. In general, if the measure of the central angle is θ , then the sector is _____ of the circle.



So, the formula for the area of a sector with central angle θ on a circle of radius r is:

Find the area of the sector above if $r = 9$ and θ is 45° .

