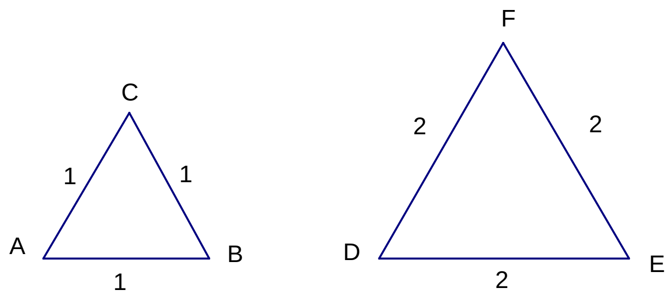
b10.4 Similar Triangles and Similar Figures

Similar figures have the same shape, but not necessarily the same size. With two similar triangles, all of the pairs of corresponding angles will be congruent, but the size of the triangles may be quite different.
For example, these two triangles are shaped the same, but one is obviously larger than the other:

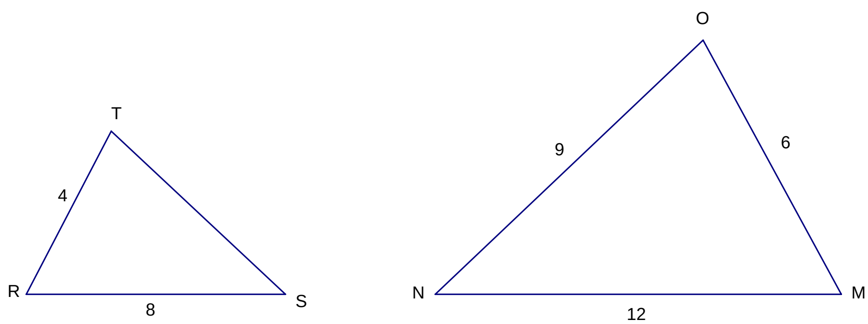


They are both equilateral triangles. All pairs of corresponding angles are congruent. So, we say that $\triangle ABC \sim \triangle DEF$.
What is the ratio $\frac{AB}{DE}$? __________ This ratio is called the scale factor: the ratio of a pair of corresponding side lengths.

Definition of Similar Triangles In order for two triangles to be similar, two things must be true:
1) All pairs of corresponding angles must be congruent, and
2) The ratios of all three pairs of corresponding sides must be equal.

So, $\triangle ABC \sim \triangle DEF$ iff $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$, and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

Now suppose you knew that these two triangles were similar, that is, $\triangle RST \sim \triangle MNO$.



1) Name the pairs of congruent angles and the pairs of corresponding sides:
    Angles ____

    Sides____

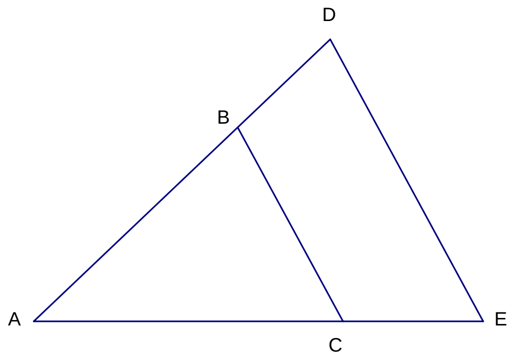
2) Find ST. ________________

To prove that two triangles are similar, you only need to show that two pairs of angles are congruent. Why?

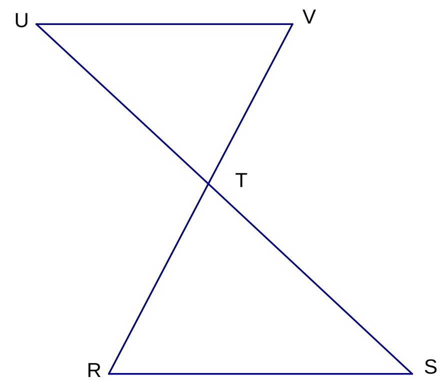
ANGLE, ANGLE (AA) PROPERTY If two angles of one triangle are congruent, respectively, to two angles of a second triangle, then the triangles are similar.

Examples:

1) Find a pair of similar triangles in each of these figures:

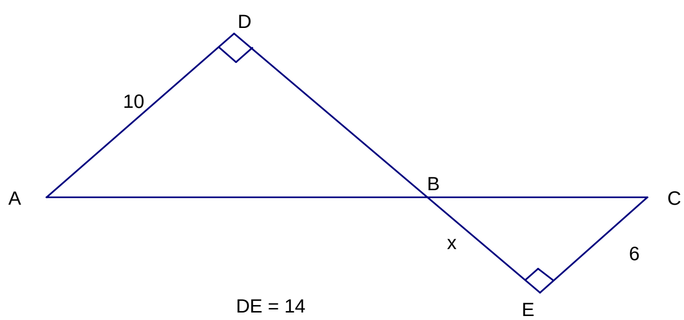


$\overline{BC} \parallel \overline{DE}$



$\overline{UV} \parallel \overline{RS}$

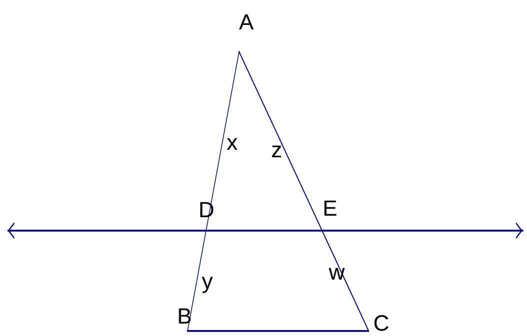
2) Find x:



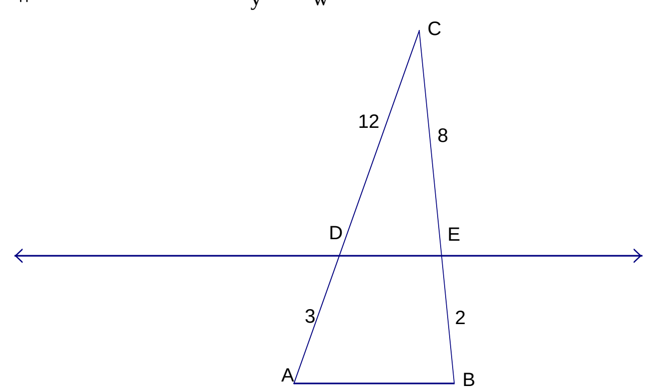
PROPERTIES OF PROPORTIONS

Theorem 10-4 If a line parallel to one side of a triangle intersects the other sides, then it divides those sides into proportional segments.

Let's show why this is true. In figure a), given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, show that $\frac{x}{y} = \frac{z}{w}$.



a)

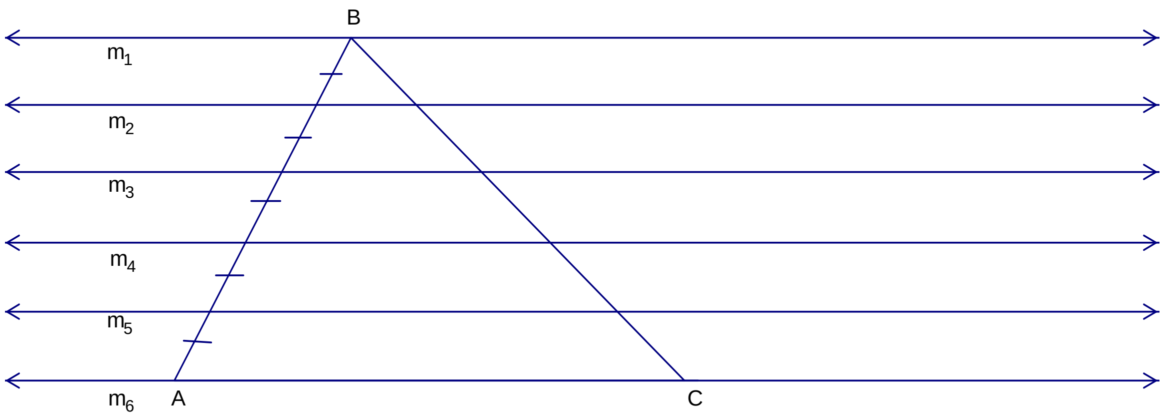


b)

The converse to this theorem is also true:
<u>Theorem 10-5</u> If a line divides two sides of a triangle into proportional segments, then the line is parallel to the third side.

Given figure b) above, is $\overleftrightarrow{DE} \parallel \overline{AB}$?

<u>Theorem 10-6</u> If parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal.



So, given that these 6 lines are all parallel, and the marked segments are congruent;
Conclusion: the segments cut off on $\overline{BC}$ are congruent to each other.

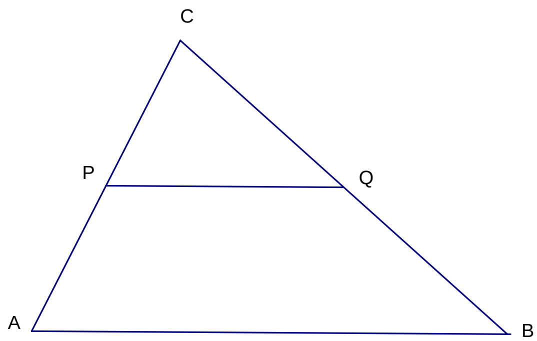
We can use this theorem to divide a line segment into any number of congruent parts.
<u>Example</u>: Divide $\overline{AB}$ into 3 congruent segments.

You can follow these steps:

1) Draw a ray $\overrightarrow{AC}$.
2) With compass, mark off 3 congruent segments on $\overrightarrow{AC}$. Label the endpoints of these segments D, E and F.
3) Construct $\overline{BF}$.
4) Now construct lines parallel to $\overline{BF}$ through points D and E.

MIDSEGMENTS OF TRIANGLES AND QUADRILATERALS

The segment connecting the midpoints of 2 sides of a triangle or 2 adjacent sides of a quadrilateral is called the midsegment. Consider $\triangle ABC$. If P is the midpoint of $\overline{AC}$ and Q is the midpoint of $\overline{BC}$, then $\overline{PQ}$ is a midsegment.



Problem: Show that $PQ = \frac{1}{2}AB$

Theorem 10-7 The Midsegment Theorem The segment connecting the midpoints of 2 sides of a triangle is parallel to the third side and half as long.

Theorem 10-8 If a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side and therefore is a midsegment.

Let's look at Example 10-12 on page 688.

INDIRECT MEASUREMENT

Similar triangles are used to do indirect measuring all the time. For instance if you needed to measure something that was very tall, such as the tree in example 10-13 on page 689, you could use the relationships between similar triangles. Let's look at that example.

Two Indirect Measurement Problems

1. Mary Christmas is looking for a Christmas tree. The tree, however can be no more than 4 meters tall. Mary finds a tree that casts a shadow of 2 m, whereas Mary (120 cm tall) casts a shadow of 0.8 m. Will the tree fit in Mary's room?

2. An observer on the shore saw a ship anchored off the coast. To find the distance from the ship to the shore, he made the measurements shown in the picture. How far is the ship from shore?