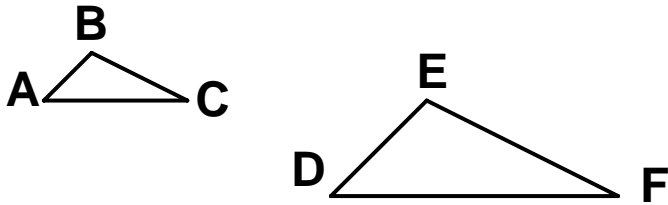


## 10.1 Congruence Through Constructions

Two figures are congruent iff they are the same size and shape.

$$\overline{AB} \cong \overline{CD} \text{ iff } AB = CD \quad \angle A \cong \angle B \text{ iff } m\angle A = m\angle B$$

Two figures are similar if they are the same shape, but not necessarily the same size. These two triangles are shaped the same, but are definitely not the same size. They are similar. We write  $\triangle ABC \sim \triangle DEF$ .



GEOMETRIC CONSTRUCTIONS The ancient Greeks constructed geometric figures using only a straightedge (with no markings) and a compass. Let's try a few constructions.

- 1. Construct a circle of radius length AB:



A circle is defined to be the set of all points in a plane that are equidistant from a given fixed point, the center. An arc of a circle is any part of the circle that can be drawn without lifting the pencil. So, the entire circle could be considered an arc. When naming an arc, we use the endpoints if there is no danger of ambiguity. Otherwise, we use 3 points. An arc that is larger than a semicircle is called a major arc; one that is smaller than a semicircle is called a minor arc.

- 2. Construct a line segment onto line  $j$  that is congruent to AB above:



Before we go to the next construction, we need to clarify what is meant by:

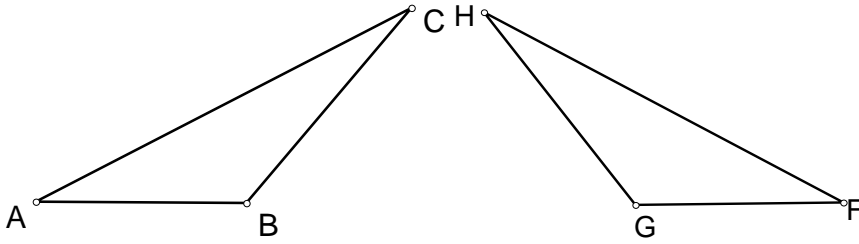
CONGRUENT TRIANGLES:  $\triangle ABC \cong \triangle A'B'C'$  iff  $\angle A \cong \angle A'$ ,  $\angle B \cong \angle B'$ ,  $\angle C \cong \angle C'$ ,  $\overline{AB} \cong \overline{A'B'}$ ,  $\overline{BC} \cong \overline{B'C'}$ , and  $\overline{AC} \cong \overline{A'C'}$ .

Corresponding Parts of Congruent Triangles are Congruent or CPCTC

Look at example 10-1 on page 648.

- a) b)

**SSS PROPERTY** (Side,side,side): If three sides of one triangle are congruent, respectively, to three sides of a second triangle, then the triangles are congruent.



Given  $\overline{AC} \cong \overline{FH}$ ,  $\overline{BC} \cong \overline{GH}$ , and  $\overline{AB} \cong \overline{FG}$ , fill in the blanks:  $\triangle$  \_\_\_\_\_  $\cong$   $\triangle$  \_\_\_\_\_.

If each student in here was to construct a triangle whose sides measured 3, 5 and 6 inches, what would be true about all of those triangles?

Look at example 10-2 on page 650. Explain in each case how the pairs of triangles are congruent.

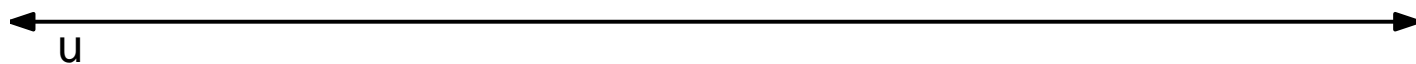
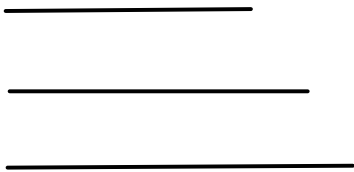
- a)
- b)

Now we are ready for 3 more constructions:

- 3-a) Construct a triangle that is congruent to  $\triangle CDE$ :



- b) Construct a triangle whose sides are congruent to these segments:



- c) Construct a triangle whose sides are congruent to these segments:

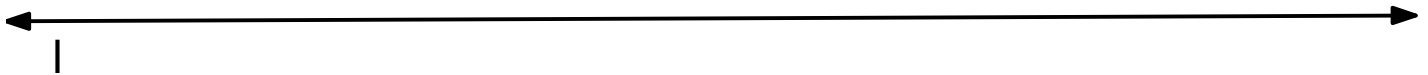
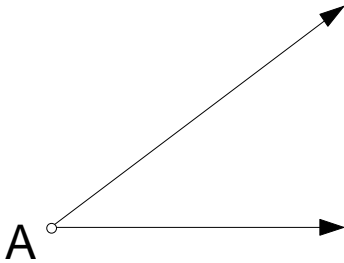


**TRIANGLE INEQUALITY PROPERTY:** The sum of the measures of any two sides of a triangle must be greater than the measure of the third side.

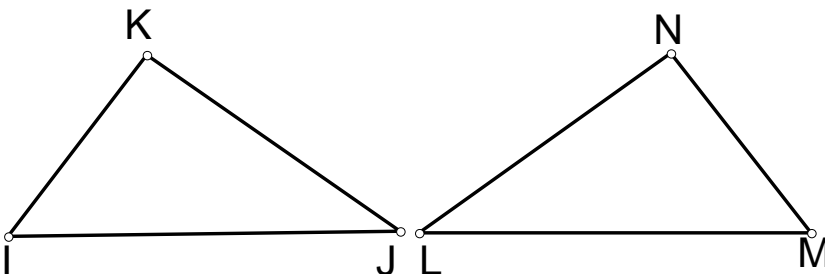
Could a triangle have sides measuring 4, 6, and 21?      What about 4, 6, and 10?

Now we are going to construct an angle:

- 4. Copy  $\angle A$  onto line  $l$ :

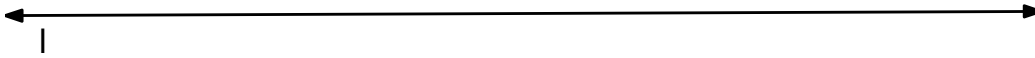
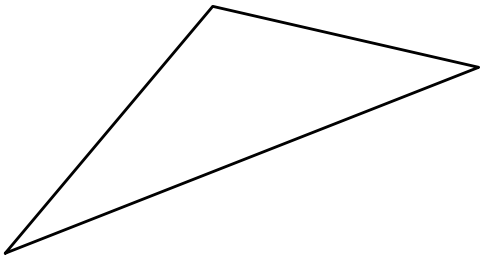


**SAS PROPERTY (Side, Angle, Side) :** If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, respectively, then the two triangles are congruent.



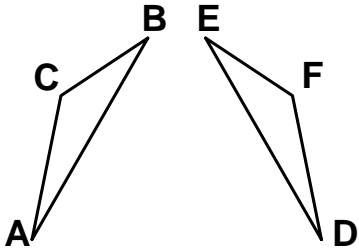
Given  $\overline{IK} \cong \overline{LN}$ ,  $\overline{IJ} \cong \overline{LM}$  and  $\angle I \cong \angle M$ , fill in the blanks  $\triangle \underline{\hspace{2cm}} \cong \triangle \underline{\hspace{2cm}}$ .

- 5. Construct this triangle by copying two sides and the included angle:

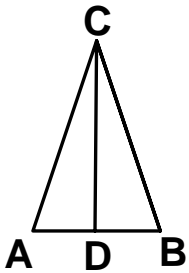


Two additional problems:

1. Given  $\overline{AC} \cong \overline{DF}$ ,  $\overline{BC} \cong \overline{EF}$ , and  $\angle C \cong \angle F$ , can you show that  $\angle A \cong \angle D$ ?

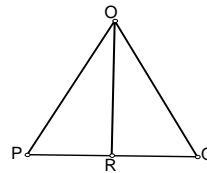


2. Given isosceles triangle ABC,  $\overrightarrow{CD}$  bisects  $\angle C$ , can you show that  $\angle A \cong \angle B$ ?



Can you show that  $\overline{AD} \cong \overline{BD}$ ?

Can you show that  $\angle ADC$  and  $\angle BDC$  are right angles?



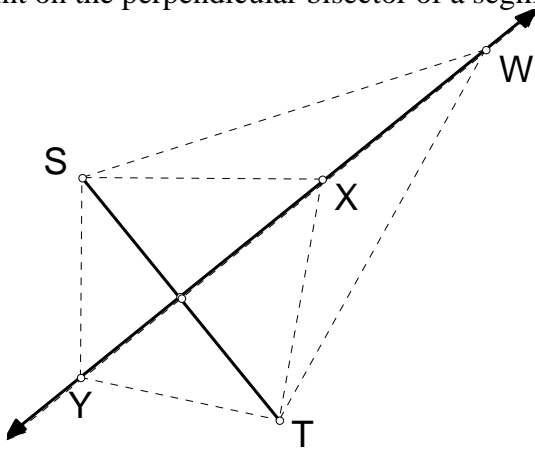
This leads us to 2 theorems:

Theorem 10-1: The following hold for every isosceles triangle:

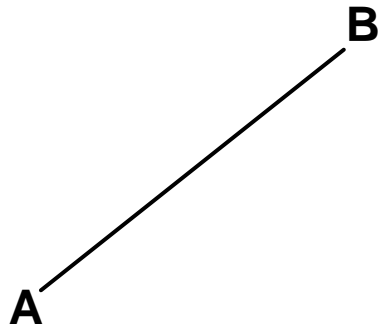
- The angles opposite the congruent sides are congruent.  
(Base angles of an isosceles triangle are congruent.)
- The angle bisector of an angle formed by two congruent sides is an altitude of the triangle as well as the perpendicular bisector of the third side of the triangle.

Theorem 10-2:

- a) Any point equidistant from the endpoints of a segment is on the perpendicular bisector of the segment.
- b) Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.



- 6. Construct the perpendicular bisector of  $\overline{AB}$ .



Earlier, we constructed a triangle using 2 sides and the included angle.

Suppose we are given 2 sides and an angle different from the included angle. Can we do the construction?