T102 SECTION 7.5 USING PERMUTATIONS AND COMBINATIONS

In this section we will learn how to count outcomes using 3 techniques:

1. Fundamental Counting Principle (A review from Chapter 2)
2. Permutations (where order matters! – that is being “first” is different than being “second”)
3. Combinations (where order is not important – just being part of the set is)

I. THE FUNDAMENTAL COUNTING PRINCIPLE

Recall that we covered this in Section 2.1, so let’s try an example to count the number of ways a multistage experiment can turn out.

EXAMPLE: You have 2 pair of slacks and 3 t-shirts. How many different outfits can you make? (Start with a tree diagram to count them.)

THE FUNDAMENTAL COUNTING PRINCIPLE:

If event M can occur in *m* ways and after it has occurred, event N can occur in *n* ways, then event M followed by event N can occur in _______________ ways.

(Note: This principle generalizes to more than just 2 stage experiments.)

EXAMPLES:

1. How many meals are possible at a restaurant where you get to choose the salad, entree and dessert, if they have 4 types of salads, 5 different entrees, and 6 yummy desserts?

2. If you are given a list of 5 TV shows and are asked to list your top 3 choices, how many ways could you do this?

3. How many different ID numbers are there if each is made up of 2 letters followed by 3 digits?

   Suppose the letter and digits cannot be repeated? ______________________________

   Suppose the first letter must be I, for Indiana, and the letters and digits cannot be repeated? _______________

4. In a horse race with 8 horses, how many different finishes (win, place, show) are possible? (THIS IS ALSO A PERMUTATION)

5. In how many ways can president, vice-president, secretary, and treasurer be chosen from a group of 10 people if no person can hold more than 1 office? (THIS IS ALSO A PERMUTATION)

II. PERMUTATIONS

A. PERMUTATIONS OF UNLIKE OBJECTS

A *permutation* is an arrangement of things in a definite order, with no repeats.

List all possible arrangements of the following sets:

Sets	Arrangements	Number of Arrangements
S = $\{a, b\}$		When n(S) = 2
S = $\{a, b, c\}$		When n(S) = 3
S = $\{a, b, c, d\}$		When n(S) = 4

Using the Fundamental Counting Principle, we can develop a formula for counting the number of permutations of *n* objects:

# of possibilities For the 1st position	# of possibilities For the 2nd position	# of possibilities For the 3rd position	. . .	# of possibilities for the n^{th} (last) position

So, if there are n objects, then the number of possible ways to arrange the objects in a row is the product of all the natural numbers from n to 1, inclusive. This expression is called *n factorial* and is denoted _________.

FORMULA FOR PERMUTING n DISTINCT OBJECTS:

If there are n objects, then there are n! ways to arrange them in a definite order with no repeats.

$$n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$$

Examples:

1. a. 6 b. $3! \cdot 4!$ c. $\frac{7!}{3! \cdot 5!}$

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2. How many ways can you arrange 5 books on a shelf?

3. How many ways could you arrange 9 crazy tweens in line waiting for Hannah Montana tickets?

This formula only works when we are arranging ALL of the objects in a certain order. But what if we only need to choose PART of the group and arrange that part in as many ways possible?

For instance, suppose that we need to choose a president, vice-president, secretary, and treasurer from a group of 10 people (no person can hold more than 1 office). How many ways can this be done? (See previous page where we used the Fundamental Counting Principle):

What you have just found is $_{10}P_4$ – the number of permutations possible from “10 items choose 4 at a time”.

FORMULA FOR PERMUTATING “n CHOOSE r” OBJECTS OR $_nP_r$ (NO LIKE OBJECTS)
If n objects are chosen r at a time, then the number of possible permutations, denoted $_nP_r$ is:
$_nP_r =$

Now, let’s choose a president, vice-president, secretary, and treasurer from a group of 10 people using the above formula.

EXAMPLES:

1. How many ways could you arrange 4 out of 9 ballerinas in a line?

2. How many ways could you arrange 7 letters of the alphabet if no letter is repeated?

3. A baseball team has 9 players. Find the number of ways the manager can arrange the batting order.

4. How many ways could you arrange the letters in the word RED?

B. PERMUTATIONS INVOLVING LIKE OBJECTS

Whenever there are "like" objects involved, we use a different formula. Let's look at an example:

List all arrangements of
the letters in the word "ZOO"?
Make a tree diagram:

List all arrangements of
the letters in the word "RED"?
Make a tree diagram:

Now, how many different ways were you able to find?

"ZOO" = _____________

"RED" = _____________

Why are they different when they both have 3 letters?

FORMULA FOR PERMUTING WITH LIKE OBJECTS (REPEATS)

If a set of n objects, of which r_1 are of one kind, r_2 are of another kind, and so on through r_k, then the number of possible permutations of all n objects is:

$$\frac{n!}{r_1!\, r_2!\, r_3! \ldots r_k!}$$

EXAMPLES: Find the number of ways the letters in the following words can be arranged:

a. BUBBLE

b. STATISTICS

III. COMBINATIONS

Combinations are just groups where the order of the objects in the group does not matter. For instance, suppose we have a club with 4 members, Al, Betty, Carol, and Dave. They need to send 2 people from the club to buy pizza. How many ways could this be done? Notice that the order of the choosing does not matter. Sending Betty and Dave is the same as sending Dave and Betty, so you would not count both of these possible choices.

Let's look at this example more closely. Let's list all possible permutations:
(How many are there? ${}_4P_2$ = ______________________ . Now list them.)

Cross out the doubles. Count the remaining groups. ___________ This is the number of combinations.

In general, when counting groups of size r chosen from n objects, we will need to divide the number of permutations by r!

FORMULA FOR THE NUMBER OF COMBINATIONS OF n OBJECTS TAKING r AT A TIME:

${}_nC_r$ =

EXAMPLES:

1. A teacher needs to choose 3 students to run an errand out of a class of 35 students. How many ways could this be done?

2. How many different 5 card hands can be dealt from a deck of 52 cards?

3. The FDA is testing a new drug with 18 volunteers. Five will receive a placebo and the rest the drug. How many different placebo groups are possible? How many different drug groups are possible?

4. What is the probability that in a 6 question True/False quiz you answer only 50% of the questions correctly**? NOTE: THIS IS THE SUM OF THE "BRANCHES" OF THE TREE DIAGRAM THAT HAVE 3 CORRECT AND 3 INCORRECT ANSWERS. WE DO NOT WANT TO CREATE THE TREE**!

a. What is the probability of answering 3 correct and 3 incorrect?____________________________

b. How many different ways are there to get 3 right and 3 wrong?___________________________
(That is, how many branches have 3 I's and 3 C's? I-C-I-C-I-C, I-I-I-C-C-C, etc… since we can't count them from the tree diagram, we can count them using our combination formula)

c. Multiply (a) by (b): ____

5. Given a class of 12 girls and 10 boys, answer the following:

a. How many committees of 5 be formed?

b. How many committees consisting of 3 girls and 2 boys be formed?

c. What is the probability that a committee of 5, chosen at random from the class, consists of 3 girls and 2 boys?

d. How many of the possible committees of 5 have no boys?

e. What is the probability that a committee of 5, chosen at random from the class, consists of only girls?

6. At the beginning of the second quarter, each of a class's 25 students shook hands with each of the other students exactly once. How many handshakes took place?

7. How many different ways can the letters M,A,T,H be arranged? What is the probability that it will spell MATH?

8. Five horses (Applefarm, Bandy, Cash, Deadbeat, and Egglegs) run in a race.
   a. In how many ways can the horses cross the finish line?

   b. In how many ways can the 1st, 2nd, and 3rd place horses be determined?

   c. Find the probability that Deadbeat finishes 1st and Bandy finishes 2nd in the race.

   d. Find the probability that the 1st, 2nd, and 3rd place horses are Deadbeat, Egglegs, and Cash, in that order.