T102 SECTION 7-4 ODDS, CONDITIONAL PROBABILITY AND EXPECTED VALUE

PART I ODDS

You hear people speak about the "odds in favor of" something happening or the "odds against" something happening all the time. What do these statements really mean?

If the odds in favor of an event are 5 to 4, this really means that out of _____ trials, it is likely to happen _____ times and to not happen _____ times.

Similarly, if the odds against an event are 6 to 9, this really means that out of _____ trials, it is likely to happen _____ times and not to happen _____ times.

Example: What would the odds be in favor of rolling a number less than 3 on the roll of a die? Since there are _____ ways to roll a number less than 3 on a die and _____ ways of not rolling a number less than 3, the odds in favor of this event are _____to_____ and the odds against this event are _____to_____.

Example: If a spinner is equally divided into 4 regions, colored red, blue, yellow, and green, what would be the odds in favor of the spinner landing on green? ___________________________

What would be the odds against the spinner landing on yellow? _______________________

Example: What would be the odds in favor of tossing a head with a fair coin? ______________________________

ODDS CAN BE WRITTEN THREE WAYS: ____

DEFINITION OF ODDS:

Let P(A) be the probability that event A occurs and $P(\overline{A})$ be the probability that A does not occur.

Then the **odds in favor** of an event A are:

$\dfrac{\text{number of ways A does happen}}{\text{number of ways A does not happen}}$ OR

And, the **odds against** an event A are:

$\dfrac{\text{number of ways A does not happen}}{\text{number of ways A does happen}}$ OR

Example: You have 8 marbles, 3 green, 4 red, and 1 white. You select a marble at random.

a. What are the odds in favor of selecting a green marble? __________________________

b. What are the odds against selecting a white marble?______________________________

Example: What are the odds in favor of selecting an Ace from a standard deck of cards? ___________________

What are the odds against selecting a diamond from a standard deck of cards?___________________

What is the probability of not selecting a diamond fro a standard deck of cards? _________________

Example: What are the odds against rain on a day where there is a 30% chance for rain?

Example: The odds against event A are 5 to 9. Find:

a. The odds for A ________________________________

b. P(A) ____

THEOREM

If the odds in favor of event E are m:n, then P(E) =

If the odds against event E are m:n, then P(E) =

Example: On an American roulette wheel, half of the slots numbered 1 through 36 are red and half are black. Two slots, numbered 0 and 00 are green.

What are the odds against a red slot coming up? ____________

What is the probability of a red slot coming up? ____________

PART II CONDITIONAL PROBABILITY

When the sample space of an experiment is affected by additional information, the "new" sample space often is reduced in size.
For instance, find the probability of having drawn an Ace from an ordinary 52-card deck, given that you KNOW that the drawn card is a heart.

What is the NEW sample space ____

What is the probability of an Ace in this NEW sample space? _____________

Example: Suppose we toss a coin 3 times. What is the probability of getting a tail on all 3 tosses given that we KNOW we got a tail of the first toss?

What is the original sample space (of tossing a coin 3 times)?

HHH, ____

What is the NEW sample space (knowing we got a tail on the 1st flip)?

____

What is the probability of getting a tail on all 3 tosses given that we KNOW we got a tail on the first toss? ______________________

Example: You toss two fair dice and examine the sum of the up faces.

What is the probability that the sum is an 11? _________________________________

What is the probability that the sum is an 11, given that you know that the sum is greater than 10? ________________

What is the probability that the sum is an 11, given that you know that the sum is prime? _______________________

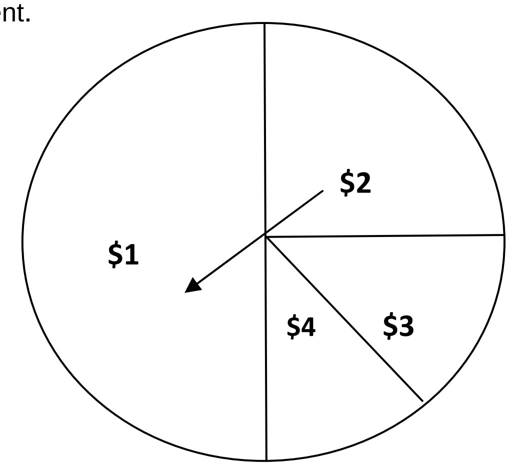
PART III EXPECTED VALUE

Expected Value is the **expected long-term average** of winnings in a game. That is, if you played **many** times, it is what you would expect to win (or lose) per trial of the game.

> **DEFINITION OF EXPECTED VALUE**
> If, in an experiment, the possible outcomes are numbers $a_1, a_2, \ldots, a_n$, occurring with probabilities $p_1, p_2, \ldots, p_n$, respectively, then the expected value E is given by the equation:
>
> E =

Expected value can be used to predict the average result of an experiment when it is repeated many times. But an expected value **CANNOT** predict the outcome of any single experiment.

Consider the spinner shown with the payoff in each sector of the circle. Using area models, we can assign the following probabilities:



P($1) = ________________ P($2) = ________________

P($3) = ________________ P($4) = ________________

Outcome				
P(Outcome)				

Determine the average payoff (or expected value) over the long-run.

Should the owner of this spinner expect to make or lose money over an extended period of time if the charge is $2.00 per spin?

> **FAIR GAMES**:
>
> A game is considered "**fair**" if and only if its expected value equals the price per game. (Nobody wins, nobody loses.)

Example: You pay nothing to play this dice game: The game is to roll a single die, and the payoff is :

$2 for rolling a 6
$1 for rolling a 5
$0 for rolling a 4
you pay $1 for rolling a 3
you pay $2 for rolling a 2
you pay $3 for rolling a 1.

Is the game fair? Use the table to help you find the expected value.

Outcome						
P(Outcome)						

Example: Ten thousand raffle tickets are sold at $2 each for a local library benefit. Prizes are awarded as follows: 2 prizes of $1,000, 4 prizes of $500, and 10 prizes of $100. What is the expected value of this raffle if you purchase 1 ticket?

METHOD 1 CONSIDERING THE $2 PURCHASE PRICE AFTER

Outcome				
P(Outcome)				

METHOD 2 CONSIDERING THE $2 PURCHASE PRICE BEFORE

Outcome				
P(Outcome)				

T102 CHAPTER 7 WORKSHEET

1. **You pay $2.50 to play this game. A spinner is equally divided into 4 sections labeled $1, $2, $3 and $4. You spin and you win the amount where the spinner stops.**
a. Find the expected value of the game.
b. EXPLAIN WHY OR WHY NOT, this is a fair game. Would you play?

2. You pay $5 to play this game. There are two urns with marbles in them. Urn 1 has 2 red, 2 blue, and 1 white, while Urn 2 has 2 red and 3 blue marbles. You select a marble from Urn 1 and place it in Urn 2. Then you select a marble from Urn 2. If the marble is red, you win $3. If it is blue, you win $6. If it is white, you win $15.
a. Find the expected value of the game.
b. EXPLAIN WHY OR WHY NOT, this is a fair game. Would you play?

3. You draw a card from an ordinary deck of cards. What is the probability that you draw a 2 knowing that you have drawn a prime numbered card.

4. In a recent election, 52% of the voters were women. For a random selection of 7 voters, find the probability of getting at least 1 woman voter. (Round to 4 decimal places