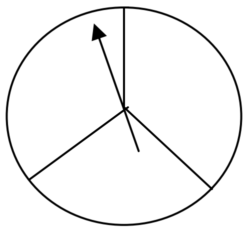
7.2 MULTISTAGE EXPERIMENTS WITH TREE DIAGRAMS
GEOMETRIC PROBABILITIES

I. TREE DIAGRAMS

Tree Diagrams are visual depictions of experiments. We will first look at a tree diagram of a **ONESTAGE EXPERIMENT** – that is, only one step.

EXAMPLE: A spinner is divided into 3 equal color regions – Red, Yellow and Blue. Model a single spin with a tree diagram.



Now, let us move on to a **TWO-STAGE EXPERIMENT**.

EXAMPLE: You are to spin the spinner twice and note the pair of colors you have spun.

Use the above diagram to answer the following:

What is the sample space?____

(**NOTE THAT EACH OF THE OUTCOMES ARE EQUALLY LIKELY – THIS IS NOT ALWAYS THE CASE**!)

What outcomes make up the event that both colors are the same? ________________________

What is the probability of getting yellow both times? __________________________________

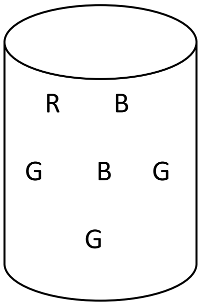
What is the probability of getting at least one red? ___________________________________

MULTIPLICATION RULE FOR PROBABILITIES FOR TREE DIAGRAMS

For all multistage experiments, the probability of the outcome along any path (branch) of the tree is equal to the product of all the probabilities along the path (branch

Now, go back and use the above property and check the probabilities of the outcomes.

EXAMPLE: You are selecting 2 marbles from a bin with 1 red, 2 blue and 3 green marbles **WITH REPLACEMENT** (you pick one, put it back, and then pick again). Construct the tree diagram.



Use the tree diagram to find the following probabilities:

a. P(both the same color) ______________________ b. P(one is blue and one is green) ______________________

c. P(both red) ________________________________ d. P(1st one is blue and the 2nd is green) _____________________

e. P(at least one is green) ______________________

EXAMPLE: This time, select 2 marbles **WITHOUT REPLACEMENT**. Construct the tree diagram and notice the change.

Use the tree diagram to find the following probabilities:

a. P(both the same color) ______________________ b. P(both red) ____

c. P(one is blue and one is green) ______________ d. P(1st one is blue and the 2nd is green) _____________________

e. P(at least one is green) ______________________

INDEPENDENT EVENTS

Two events are independent when the outcome of the first has no influence on the outcome of the second.

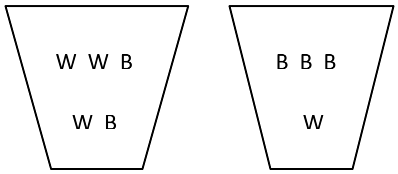
For any independent events, E_1 and E_2,

$$P(E_1) \cap P(E_2) = P(E_1) \cdot P(E_2)$$

In the above examples involving the marbles, which involved INDEPENDENT EVENTS?

Can you think of other events that are typically independent?

EXAMPLE: You have 2 urns. Urn 1 contains 3 white and 2 black marbles. Urn 2 contains 1 white and 3 black marbles. The experiment is to draw a marble from Urn 1, note its color and place it in Urn 2. Then draw a marble from Urn 2 and note its color.



Tree Diagram:

Find: P(both marbles are the same color) ________________________________

P(the marble drawn from Urn 2 is white) _________________________________

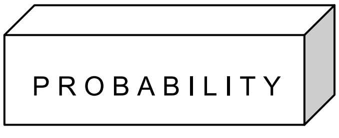
P(at least one marble drawn is white) __________________________________

P(neither of the marbles is white) ________________________________

NOTE THAT THE LAST TWO QUESTIONS ARE COMPLEMENTS OF EACH OTHER. DO THEIR PROBABILITIES SUM TO 1?

EXAMPLE(#5) A committee consists of 10 members: 4 women and 6 men. Three members are selected at random to be sent to a meeting in Hawaii. The three names were drawn out a hat and all three were women's names. What is the probability of such luck?

EXAMPLE: A box contains all the letters in the word PROBABILITY. Suppose 4 letters are randomly drawn one by one without replacement. What is the probability of the outcome BABY, in that exact order? (HINT: Construct only that branch of the tree)



EXAMPLE: (#8) An assembly line has two inspectors. The probability that the first misses a defective item is .05. If the defective item passes the first inspector, the probability that the second inspector will miss it is .01.

a. What is the probability that a defective item will pass by both inspectors? ____________________________

b. What is the probability that a defective item will not pass by both inspectors? _________________________

PROBLEMS INVOLVING THE PHRASE "AT LEAST ONE"

In these cases it is often more useful to take advantage of the complement. Since "at least one" is the complement of "none", we can say:

P(at least one) = 1 - P(none) **[Since $P(A) = 1 - P(\overline{A})$]**

EXAMPLE: (#26) A husband and wife discover that there is a 10% probability of passing on a hereditary disease to each of their children. If they plan to have 3 children, what is the probability that at least one child will inherit the disease?

II. MODELING GAMES

We can use models to analyze games that involve probability. For instance: Bill and Mary play a game where there are two red marbles and one white marble in a box. Mary mixes the marbles, and Bill draws two marbles at random without replacement. If the two marbles match, Bill wins; other wise Mary wins. Does each player have an equal chance of winning?

III. GEOMETRIC PROBABILITY

Geometric probability (also known as an AREA MODEL) uses geometric shapes to help determine probabilities. The geometric region represents the sample space and the shaded area(s) the event(s) in question. Consider the following dart board.

What is the probability of hitting any non-white region? ________________________________

What is the probability of hitting a gray region? ____________________________________