# T102 SECTION 7-1 HOW PROBABILITIES ARE DETERMINED

## I. INTRODUCTION

Probability, with its roots in gambling, is used in areas such as weather, sports, politics, insurance, etc.

Probabilities are **RATIOS** expressed as fractions, decimals, or percents, and they are determined by considering the results or outcomes of experiments.

## II. ACTIVITY "Flipping A Coin"

When you flip a coin, what are the possible outcomes? _____________ or_______________

Flip the coin 20 times. How many heads did you personally toss? ____________

The ***relative frequency*** is obtained by dividing the number of heads **actually observed** by the total number of tosses. Find the *relative frequency for the number of heads observed in your 20 tosses* Round to 3 decimal places if needed.

Let us combine our data. The total number of heads for the entire class is _____.
The total number of tosses for the entire class was _____.
So, the *relative frequency for the number of heads observed by the class*: ________________

## III. DEFINITIONS

**EXPERIMENT**: An activity where the results can be observed and recorded
**OUTCOME**: Any possible result of an experiment
**SAMPLE SPACE**: The set of ALL possible outcomes for an experiment
**SIZE**: The cardinal number of the sample space.

| **EXPERIMENT** | **Outcomes** | **Sample Space** | **Size** |
|---|---|---|---|
| Tossing a Coin | Heads or Tails | A= {Heads, Tails} | n(A) = |
| Rolling a Die | | | |
| Having 3 Children | | | |
| Drawing Cards | 2♥, 3♥, ... Q♥, K♥, A♥<br><br>2♣, 3♣, ...A♣<br><br>2♦, 3♦, ... A♦<br><br>2♠, 3♠, ... A♠ | | |
| Tossing 2 Coins | | | |

**EVENT**: Any Subset of the Sample Space of an Experiment

| EXPERIMENT | SIZE | EVENT | SAMPLE SPACE OF THE EVENT | SIZE |
|---|---|---|---|---|
| Rolling a Die | | "Rolling an Even Number" | | |
| Drawing a Card from a Deck | | "Drawing a Face Card" | | |
| Tossing 2 coins | | "Getting at Least One Tail" | | |

## IV. DETERMINING PROBABILITIES

There are two main schools of thought in determining probabilities:

1. Empirical
2. Theoretical

**A. EMPIRICAL (OR EXPERIMENTAL) METHOD**: An experiment is actually repeated many times, we observe the outcomes, and the relative frequencies are determined. (Like the coin toss!)

The probability of an outcome occurring is: ______________________________________________

and only suggests what will happen in the "long run".

**EXAMPLE**: Suppose for an experiment, S = {rock, scissors, paper} and the following table lists the number of times each of these outcomes occurred. Complete the table.

| OUTCOMES | NUMBER of OCCURENCES | RELATIVE FREQUENCY |
|---|---|---|
| ROCK | 295 | |
| SCISSORS | 301 | |
| PAPER | 304 | |
| Total # of Tries | | |

**B. THEORETICAL METHOD**: This method is much more practical because we do not actually have to DO the experiment and collect the data. Consider the situation where outcomes are EQUALLY LIKELY. This means that they all have the same chance of occurring. In this case, the theoretical probability of a single outcome occurring is:

Examples: P(rolling a 2 with a die) =

P(tossing a tail with a coin) =

P(drawing the Queen of Hearts) =

**PROBABILITY OF AN EVENT WITH EQUALLY LIKELY OUTCOMES**

For an experiment with sample space S and equally likely outcomes, the *probability of an event* A is given by:

$$P(A) = \frac{n(A)}{n(S)} =$$

**Example**: Let the sample space of an experiment be: S = {1,2,3,4,5,...,25}. If a number is chosen at random, calculate each of the following probabilities: n(S) = ______

a. The event A that an even number is drawn.

n(A) = _________ So, $P(A) = \frac{n(A)}{n(S)} =$

b. The event B that a number less than 10 and greater than 20 is drawn.

c. The event C that a number less than 26 is drawn.

d. The event D that a prime number is drawn.

e. The event E than a number both even and prime is drawn.

**PROPERTY PROBABILITY OF AN EVENT**

The ***probability of an event*** is equal to the sum of the probabilities of the events representing all the possible outcomes of that event

**Example**: If we draw a card at random from an ordinary deck of playing cards, what is the probability that:

**<u>Method 1</u>** **<u>Method 2</u>**

a. The card is an ace?

b. The card is an ace or a queen?

## V. MUTUALLY EXCLUSIVE EVENTS (Disjoint or No intersection)

Events that have no outcomes in common. Let's list some examples of mutually exclusive events. You are given the experiment. Write events within that experiment that are mutually exclusive.

| EXPERIMENT | EVENT A | EVENT B | EVENT C |
|---|---|---|---|
| **Rolling a Die** | | | X |
| **Tossing a Coin** | | | X |
| **Picking a number out of a Hat (numbers 1-15)** | | | |

**Note**: Given two (could always be more) mutually exclusive events A and B

$A \cap B =$ ________ (Read $\cap$ as "and")

$n(A \cup B) =$ ________ + ________ (Read $\cup$ as "or")

$P(A \cup B) =$ ________ + ________

## VI. NON-MUTUALLY EXCLUSIVE EVENTS (There is an Intersection)

These are events that have some outcomes in common. For example, when drawing a card, the events might be drawing a heart and drawing an ace. For this, we need a different formula to find the probability of their union.

**UNION RULE FOR EVENTS**

$P(A \cup B) =$

and if the events are equally likely: $P(A \cup B) =$

**Example**: If A is defined as drawing a face card and B is defined as drawing a heart,

(a) list the outcomes in A

(b) list the outcomes in B

(c) are A and B mutually exclusive?

(d) list the outcomes in $A \cap B$

(e) find $P(A \cup B)$ Read as: "the probability of drawing a face card OR a heart"

## VII. COMPLEMENTARY EVENTS (Just like complementary sets!)

Two mutually exclusive events whose ***union is the sample space*** are ***complementary events***. That is, $\overline{A}$ contains all of the outcomes of S that are *not in A*.

$$\text{So, } A \cup \overline{A} = S$$

Note the difference and similarity between mutually exclusive (can be 2 or more events) and complementary (only 2 events). Thus, all complementary events are mutually exclusive, but not all mutually exclusive events are complementary.

List some examples of two complementary events:

**PROPERTY OF COMPLEMENTARY EVENTS**

If A is an event and $\overline{A}$ is its complement, then

$P(A) + P(\overline{A}) =$ ______ so, $P(\overline{A}) =$ ______________________ and, $P(A) =$

## VIII. PROPERTIES OF PROBABILITY (REVIEW)

1. $P(\varnothing) =$ ________ (Impossible event)
2. $P(S) =$ ________, where S is the entire Sample Space (Certain event)
3. For any event A, _____ $\leq P(A) \leq$ ______
4. A and B are events and $A \cap B = \varnothing$, then $P(A \cup B) =$ __________________(Mutually Exclusive events)
5. If A and B are events, then $P(A \cup B) =$ _______________________(Non-Mutually Exclusive events)
6. If A is an event, then $P(\overline{A}) =$ _____________________ (Complementary Events)