T102 SECTION 6-3 NONTERMINATING DECIMALS

RECALL: Any rational number can be expressed as a decimal – either terminating or repeating.

Given the rational number $\frac{a}{b}$ $\frac{a}{b}$ in simplest form (reduced):

1. If the prime factorization of *b* contains only ______'s and/or _______'s then $\frac{a}{b}$ can be written as a _________________________ decimal.

2. If the prime factorization of *b* contains factors other than ______'s and/or _______'s then $\frac{a}{b}$ will be written as a _________________________ decimal.

Consider $\frac{9}{40}$ Does it terminate? _______________ Change to a decimal.

<u>Method 1: Make denominator a power of 10</u> **<u>Method 2: Division</u>**

Consider $\frac{3}{11}$ Does it terminate? _______________ Change to a decimal.

This is an example of a ***repeating decimal***, and the repeating block of digits is called the ***repetend.*** We write this as _______________ where the bar indicates that the block of digits underneath is repeated continuously.

EXAMPLES: Change each of the following to a decimal.

$\frac{5}{6}$ $\frac{3}{8}$

EXAMPLES: Change each of the following to a decimal

$\frac{5}{11}$

$\frac{1}{7}$

FOOD FOR THOUGHT: As we went through the division process in the final example $\frac{1}{7}$, we obtained remainders of 3, 2, 6, 4, 5, and 1. These are ***all the*** possible nonzero remainders when dividing by 7– that is 6 possible remainders. (If we got a 0, it would have terminated. Anything larger than 6 meant we didn't divide correctly.). Since each of the possible remainders had occurred, at this point, one of the possible remainders ***must*** reoccur in the next subtraction. This is where the repetend for $\frac{1}{7}$ must begin again (if not before) and it did! In general, if $\frac{a}{b}$ is any rational number in simplest form and does not terminate, the repetend will have at most ________________ digits.

Now, let's try one a bit more challenging. Using the above information, how many digits are possible in the

repetend of $\frac{1}{17}$? __________

Now, let's divide – using our calculator and a new method to help us since we know the repetend could possibly be quite lengthy.

$\frac{1}{17}$ =

You try: $\frac{1}{19}$

WRITING A REPEATING DECIMAL AS A FRACTION

Recall: .62 = =

But when we have a repeating decimal we have infinitely many digits, so there is no single power of 10 that can be place in the denominator. To overcome this difficulty, we must somehow eliminate the *infinitely repeating* part of the decimal by multiplying by a power of 10 *and use a little bit of algebra*.

EXAMPLES:

$.\overline{7}$

$.\overline{45}$

$.3\overline{24}$

$2.3\overline{45}$

$.\overline{9}$ THIS IS A VERY SPECIAL REPEATING DECIMAL.

ORDERING REPEATING DECIMALS

Line up the numbers vertically with decimal points aligned. Extend the repetend out a few times. Compare.

Order from least to greatest $5.\overline{2138}$ $5.213\overline{8}$ $5.21\overline{38}$ $5.2\overline{138}$

Find three rational numbers between $.\overline{35}$ and $.\overline{351}$