T102 SECTION 6-1 INTRODUCTION TO DECIMALS

In this section, we will explore relationships between fractions and decimals and see how decimals are an extension of the base-ten number system. The word decimal comes from the Latin decem, meaning “ten”.

RELATING TO MONEY

One of everyone’s first encounters with a decimal is in the context of money. A child sees the price of an IPOD Shuffle is $79.84. Let’s look at different ways to express this money amount.

$79.84 = “____” (words)

$79.84 = ______________________ Because $0.84 is __________________of a dollar.

$79.84 = __________ dollars + __________ dimes + __________ pennies

__________ + _______ . ____________ + _____________ . _______________

Because a dime is _____________ of a dollar and a penny is _________________ of a dollar.

EXPANDED NOTATION

$79.84 = __________ + __________ + ________ . _________ + ____________ . ________________

= _____________ + _____________ + _____________ + _____________

READ THE FOLLOWING NUMBERS. THEN WRITE IN EXPANDED NOTATION USING POWERS OF 10.

12.659 = ____

409.0783 = ____

0.54321 = ____

2,084.08 = ____

CONCRETE REPRESENTATIONS

We can use Base-10 blocks to help model decimals. However, we might need to change what each block represents from problem to problem. The unit block should always represent the “smallest place” in the number. Draw the Base-10 block representation of each.

1.35

1.204

CONVERTING FRACTIONS TO DECIMALS

1**. DENOMINATOR IS A POWER OF 10**
Read the fraction and write as a decimal.

$\frac{7}{10}$ $\frac{39}{100}$ $\frac{9}{1000}$ $\frac{2.94}{100}$

2. **DENOMINATOR IS NOT A POWER OF 10**

If the denominator is not a power of 10, as in $\frac{3}{5}$, we can convert it so it does.

$\frac{3}{5} = \frac{\quad}{10}$ which is ________ (as a decimal)

Before we take this further, let's look at the powers of 10 and see what they have in common. Write the prime factorization of each of the following:

10 100 1,000 10,000

What do we see in common?

How is the exponent(s) related to the number of zeros in the original number?

Using what we just concluded above, we can easily change the following fractions to decimals.

$\frac{3}{5^2}$ $\frac{1}{8}$

$$\frac{7}{2^3 \cdot 5^4} \qquad\qquad \frac{4}{250}$$

Notice that in all the above examples we were able to change the denominator to some power of 10. The original denominators were all combinations of 2's and/or 5's and we simply incorporated in through multiplication enough 2's or 5's to make a power of 10. They are all examples of ***terminating decimals – decimals that can be written with a finite number of places to the right of the decimal point.***

But what about: $\frac{2}{3} = \frac{2 \cdot ?}{3 \cdot ?}$ What can we multiply 3 by to get a power of 10?

(This is an example of a decimal that does not terminate.)

The leads us to the following theorem:

THEOREM A rational number *in simplest form* can be written as a terminating decimal if, and only if, the prime factorization of the denominator contains no primes other than 2 or 5.

NOTE: To determine whether a rational number can be represented as a terminating decimal, we only need to consider the prime factorization of the denominator, BUT **The fraction must be in simplest (reduced) form first**.

Which of the following can be written as a terminating decimal? Explain why or why not.

$$\frac{7}{8} \qquad \frac{21}{28} \qquad \frac{37}{768} \qquad \frac{11}{250} \qquad \frac{1}{75}$$

<u>ORDERING TERMINATING DECIMALS</u>

1. Line up the numbers by place value
2. Start at the left and find where the digits differ
3. Compare these digits. The number with the larger digit is greater
4. **NEGATIVES ARE IN REVERSE**

0.543 _______ 0.563 2.765 _______ 3.765 0.0087 _______ 0.087

-2.34 _______ -2.35 43.566 _______ 34.566

Order from greatest to least:

3.45 3.35 3.42 3.451 3.358 3.405