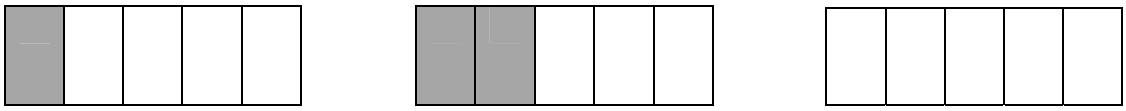
T102 SECTION 5.2 THE SET OF RATIONAL NUMBERS

I. ADDITION WITH "LIKE" DENOMINATORS

DEFINITION OF ADDITION OF RATIONAL FRACTIONS

If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$



$\frac{1}{5}$ + $\frac{2}{5}$ =

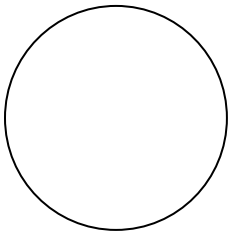
So, $\frac{1}{5} + \frac{2}{5} =$

0 1

II. ADDITION WITH "UNLIKE" DENOMINATORS

But what if the fractions do not have the same denominator?

For instance, $\frac{1}{3} + \frac{1}{4}$



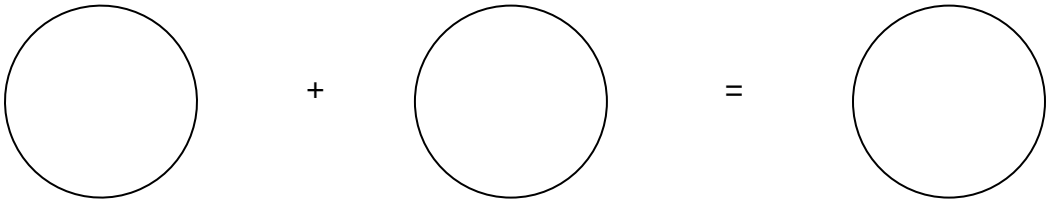
But, how do we count this? We need to find a way to combine the two drawings to find the sum. Let's "build-up" each fraction:

$\frac{1}{3} = \frac{\_}{6} = \frac{\_}{9} = \frac{\_\_}{12} = \frac{\_\_}{15}$

Now, compare these two lists and look for a "like"

denominator. What is it? _____

$\frac{1}{4} = \frac{\_}{8} = \frac{\_\_}{12} = \frac{\_\_}{16} = \frac{\_\_}{20}$



AN INTERESTING PROPERTY

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

EXAMPLES: **Using the LCD** **Using the Above Property**

$\frac{2}{3}+\frac{1}{5}$

$\frac{3}{4}+\frac{-2}{5}$

$\frac{3}{x}+\frac{4}{y}$

$\frac{1}{15}+\frac{-2}{21}$

III. MIXED NUMBERS

Mixed numbers are numbers that are the **sum** of an integer and a fractional part of an integer. For example, if a nail is $2\frac{3}{4}$ inches long, this means 2 inches **plus an additional** $\frac{3}{4}$ inches. (It is common to think that since 2y means 2 times y, that $2\frac{3}{4}$ means 2 times $\frac{3}{4}$, but this is **incorrect**!)

Change the following **mixed numbers to improper fractions**.

Using the Coventional Algorithm

$6\frac{1}{4}$

Change with Meaning

$6\frac{1}{4} = 6+\frac{1}{4} =$

$-5\frac{3}{7}$ $-5\frac{3}{7}$

Change the following **improper fractions to mixed numbers**.

Using the Coventional Algorithm Change with Meaning

$\frac{13}{4}$ $\frac{13}{4} = \frac{4+4+4+1}{4} =$

$\frac{-29}{6}$ $\frac{-29}{6}$

IV.

PROPERTIES OF ADDITION FOR RATIONAL NUMBERS

Given any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ where, *b* and *d* are non-zero integers:

1. Closure
2. Commutative
3. Associative
4. Additive Identity
5. Additive Inverse — For any rational number $\frac{a}{b}$ there exists a unique number _____ such that:

Name the additive inverse of the following:

$\frac{5}{6}$ $\quad -\frac{2}{5}$ $\quad -\left(-5\frac{3}{7}\right)$

V. ADDITION OF MIXED NUMBERS (Know how to add using the given mixed numbers)

$2\frac{3}{8}+5\frac{5}{6}$ $\quad \begin{array}{r} 2\frac{3}{8} \\ +5\frac{5}{6} \\ \hline \end{array}$ $\quad 82\frac{9}{25}+17\frac{32}{50}$

VI. SUBTRACTION OF RATIONAL NUMBERS

SUBTRACTION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

AN INTERESTING PROPERTY

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$\frac{15}{28} - \frac{11}{28} =$ $\quad \frac{3}{4} - \frac{3}{8} =$

SUBTRACTION OF MIXED NUMBERS (Know how to subtract using the given mixed numbers)

$5\frac{1}{3}-2\frac{3}{4}$ $\quad \begin{array}{r} 5\frac{1}{3} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$ $\quad 12\frac{2}{9}+-15\frac{5}{6}$

VII. ESTIMATION WITH RATIONAL NUMBERS

Many times when estimating with fractions, it is helpful to round to a *convenient fraction* –

for instance: $0,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{2}{3},\ \frac{3}{4},\ \text{or}\ 1$

For example, if you got 59 out of 80 questions correct on your test, this is about $\frac{60}{80}\ \text{or}\ \frac{6}{8}\ \text{or}\ \frac{3}{4}$

Then we can conclude that $\frac{3}{4}$ is a HIGH ESTIMATE

(You actually got less than $\frac{3}{4}$ of the test correct, since 59 < 60, then $\frac{59}{80} < \frac{60}{80} = \frac{3}{4}$).

Approximate each of the following using $0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4}, \text{or}\ 1$.. Tell if your estimate is low or high.

$\frac{3}{197}$ $\frac{8}{9}$ $\frac{47}{64}$ $\frac{2002}{2000}$ $\frac{62}{31}+\frac{99}{98}$