T102 CHAPTER 5
SECTION 5.1 THE SET OF RATIONAL NUMBERS

I. INTRODUCTION

The set of rational numbers, denoted Q, is the set of all numbers of the form $\frac{a}{b}$ where a and b are integers and b $\neq$ 0.

The number a is called the ____________________

and the number b is called the _____________________.

We can write this using set builder notation as: Q = $\{\ \_\_\_\_\_\_\_\ |\ \ \}$

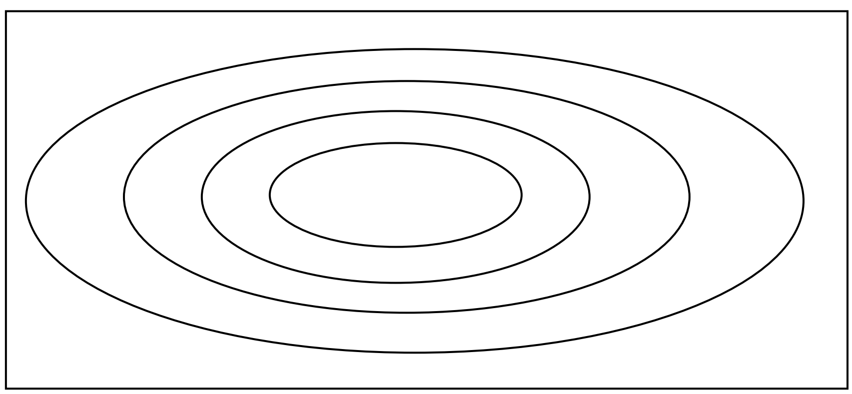
A <u>fraction</u> is any number of the form $\frac{a}{b}$ where a and b are any numbers (not necessarily integers), with b $\neq$ 0. (Note that all rational numbers are fractions, but not all fractions are rational numbers).

Examples: Classify as fraction, rational number or both.

$\frac{-4}{7}$ $\qquad$ $\frac{-1}{\sqrt{2}}$ $\qquad$ $\frac{0}{7}$ $\qquad$ -5

VENN DIAGRAM:
OF THE REAL
NUMBERS

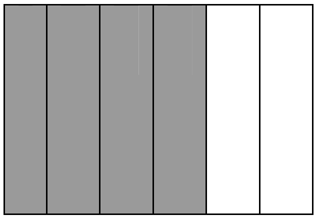
N: Natural Numbers
W: Whole Numbers
I: Integers
Q: Rational Numbers
R: Real Numbers

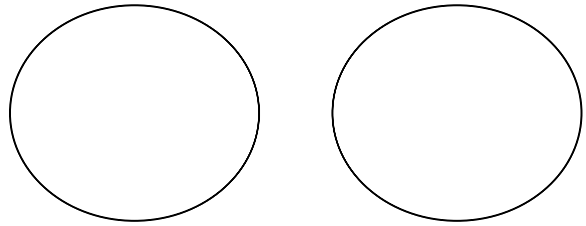


TRUE OR FALSE: $I \subseteq Q$ $\qquad$ $W \subseteq I$ $\qquad$ $I \subseteq N$ $\qquad$ $N \subseteq W$ $\qquad$ $N \subseteq Q$ $\qquad$ $W \subseteq Q$

II. **MODELS FOR FRACTIONS**

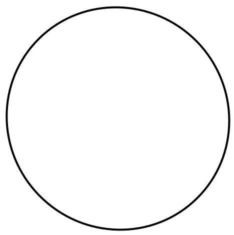
A. COLORED REGION (or AREA) MODEL



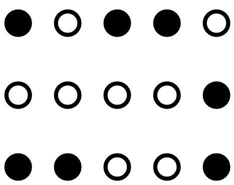


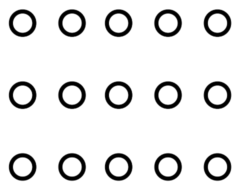
What fraction is shaded? ___________ What fraction is shaded? ___________

Shade ¾.



B. SET MODEL





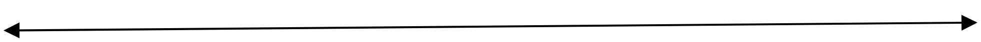
What fraction of the balls are shaded? _________ Shade 1/3 of the balls.

C. NUMBER LINE MODEL

Shade 4/5 of the unit segment |--------|--------|--------|--------|--------|
0 1

Place the following rational numbers on a number line at the proper intervals:

$$2\ ,\ \frac{3}{4}\ ,\ -\frac{3}{2}\ ,\ 0\ ,\ -\frac{3}{4}\ ,\ -1\ ,\ -\frac{5}{4}\ ,\ 1\ ,\ \frac{7}{4}\ ,\ -2$$



D. FRACTION STRIP MODEL (See page A-29 in the Activities Book)

Find strips that can be folded into parts so that the resulting strip is equal in length to the given fraction. Folds may be made ONLY on the lines of the strips.

$\frac{1}{2}$ = = = $\frac{6}{8}$ = =

Proper Fractions

A fraction $\frac{a}{b}$ where $0 \le |a| < |b|$

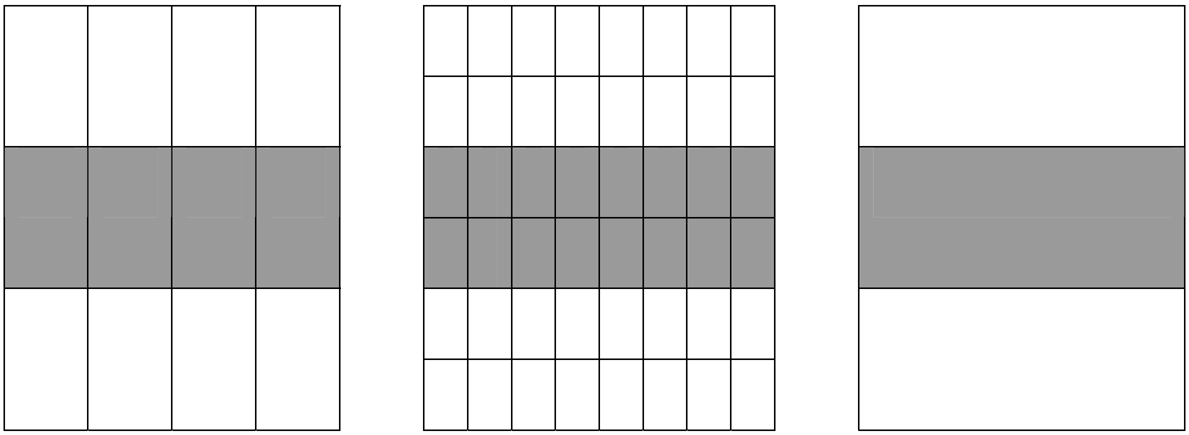
Examples:

Improper Fractions

A fraction $\frac{a}{b}$ where $|a| \ge |b| > 0$

Examples:

III. **EQUIVALENT OR EQUAL FRACTIONS**



$$\frac{\ }{12} = \frac{\ }{48} = \frac{\ }{3}$$

Notice that the *value* of the fraction does not change if its numerator and denominator are multiplied (or divided) by the same nonzero whole number.

FUNDAMENTAL LAW OF FRACTIONS

Let $\frac{a}{b}$ be any fraction and n a nonzero whole number, then

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}$$

We use this law to “build up” fractions (ie. common denominators) and also to simplify (ie. reduce) them.

Find the value for x such that :

$$\frac{12}{15} = \frac{x}{45}$$

$$\frac{12}{x} = \frac{1}{5}$$

IV. SIMPLIFYING FRACTIONS

DEFINTION OF SIMPLEST FORM

A rational number $\frac{a}{b}$ is in simplest form if a and b have no common factor greater than 1, that is, a and b are relatively prime (that is, the GCD (a, b) = 1).

$$\frac{28}{56} \qquad \frac{-27}{32} \qquad \frac{28ab^2}{42a^2b^2}$$

$$\frac{(a+b)^2}{3a+3b} \qquad \frac{x^2+x}{x+1} \qquad -\frac{7x}{15y}$$

$$\frac{3+x^2}{3x^2} \qquad \frac{3+3x^2}{3x^2} \qquad \frac{a^2-b^2}{a+b}$$

V. EQUALITY OF FRACTIONS

We can use three methods to determine if two fractions are equal.

A. Simplify (reduce) both fractions

$$\frac{12}{42} \qquad \frac{10}{35}$$

B. Rewrite both fractions using the *Least Common Denominator* (the LCM of the denominators)

$\frac{15}{24}$ $\frac{10}{16}$

C. Rewrite both fractions with *any common denominator*

$\frac{27}{72}$ $\frac{15}{40}$

This last method leads us to the following property:

PROPERTY	Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal if, and only if ad = bc

Use the above property to determine if the following fractions are equal:

$\frac{14}{63}$ and $\frac{10}{45}$ $\frac{17}{27}$ and $\frac{25}{45}$

VI. ORDERING FRACTIONS

A. Like Denominators: It is simple to compare fractions that have like denominators; just compare the numerators. The one with the greater numerator is the greater fraction.

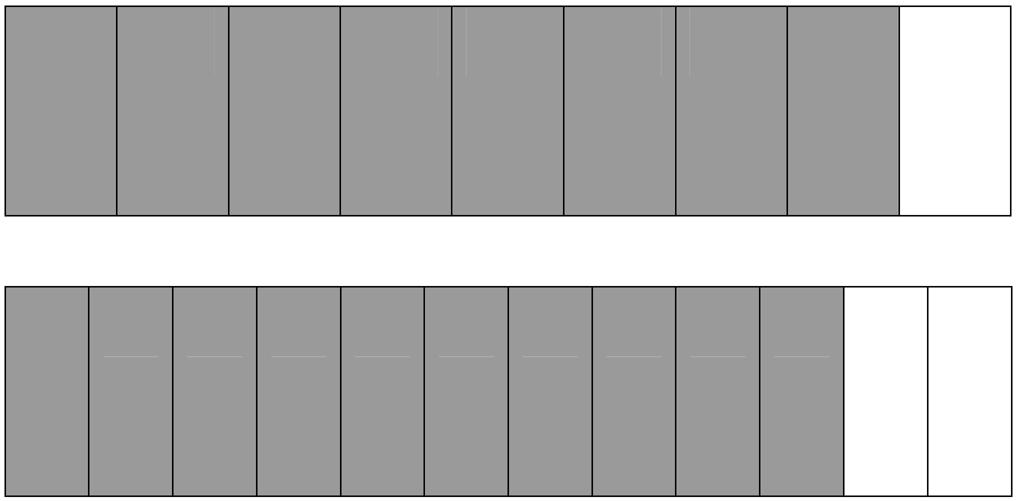
Insert <, >, or =.

$\frac{4}{11}$ ☐ $\frac{8}{11}$ $\frac{12}{22}$ ☐ $\frac{10}{22}$

B. Unlike Denominators

It is a bit more tricky when the denominators are not the same.

1. **1. AREA MODEL** $\frac{8}{9}$ □ $\frac{10}{12}$



2. MAKE SAME DENOMINATORS AND COMPARE

$\frac{3}{5}$ and $\frac{5}{9}$

$\frac{-2}{3}$ and $\frac{-1}{2}$

$\frac{51}{50}$ and $\frac{39}{41}$

3. USING CROSS-PRODUCTS

THEOREM	If a, b, c, and d are integers and b $> 0, d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if, and only if ad $> bc$

$\frac{3}{7}$ and $\frac{5}{12}$

$\frac{-9}{12}$ and $\frac{-4}{5}$

EXAMPLE: Order the following fractions from least to greatest:

$$\frac{3}{4}, \frac{9}{16}, \frac{5}{8}, \text{and } \frac{2}{3}$$

VII. DENSENESS OF RATIONAL NUMBERS

DENSENESS PROPERTY FOR RATIONAL NUMBERS

Given rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ **,** there is another rational number between these two numbers.

Find one rational number between $\frac{2}{3}$ and $\frac{1}{2}$.

.

Find ***two*** rational numbers between $\frac{7}{18}$ and $\frac{1}{2}$.

.

QUESTION? How many rational numbers lie between ¼ and ¾ (or between **any** two rational numbers)?