

## M119 – Practice Test 3

1) Evaluate using properties of logarithms (**DO NOT USE A CALCULATOR**):

a)  $\ln\sqrt{e}$

b)  $e^{2\ln 3}$

c)  $e^{3 \ln 2 - 2 \ln 5}$

d)  $\ln \frac{e^3\sqrt{e}}{e^{1/3}}$

**Convert to a logarithmic equation.**

2) a)  $e^{-3} = \frac{1}{64}$

b)  $5e^x = 2$

3) Convert from log form to exponential: a)  $\ln x = 1.6094$

b)  $\ln 39 = 3.6636$

**Express as a single logarithm and simplify, if possible.**

4)  $\ln 42 - \ln 6$

5)  $\ln x^2 - 2 \ln\sqrt{x}$

6)  $\ln 17x + 2(\ln x - \ln y)$

7)  $\ln w - 3[2\ln(w - 5) - \ln(w + 5)]$

8)  $\frac{2}{3}[\ln(y^2 - 144) - \ln(y + 12)] + \ln(y + y)$

**Solve the exponential equation algebraically.**

9) a)  $3x^2 + 5x = \frac{1}{81}$

b)  $27 = 3^{5x} \cdot 9x^2$

10) a)  $1000e^{4t} = 90$

b)  $8 - 2e^{0.04x} = 0$

**Solve the logarithm equation algebraically.**

11) a)  $\ln x = 17$

b)  $\ln(6x - 1) = \ln 3 - \ln(x - 3)$

**Differentiate.**

12) a)  $y = 4e^{x^2}$

b)  $y = e^{3x^2} + x$

**Find the derivative.**

13) a)  $y = 5x^2e^{3x}$

b)  $y = (x^2 - 2x + 3)e^x$

14) a)  $y = \frac{e^x}{3x^2 + 7}$

b)  $y = (e^{x^3} - 1)^5$

**Find the derivative of the function.**

15) a)  $y = \ln(7 + x^2)$

b)  $y = \frac{\ln x}{x^5}$

**Find the derivative.**

16) a)  $y = \frac{e^x}{\ln x}$

b)  $f(x) = \ln(e^{4x} - 4)$

17) a)  $f(x) = e^{8x} \ln x$

b)  $y = \ln \sqrt{\frac{x^2 + 1}{2x + 3}}$

**Differentiate.**

18)  $f(t) = \ln [(t^3 - 5)(t^6 + 1)]$

**Find the indicated tangent line.**

19) Find the tangent line to the graph of  $f(x) = 2e^{4x}$  at the point (0, 2).

**Solve.**

- 20) How long will it take for the population of a certain country to triple if its annual growth rate is 1.1%? Assume the population is growing exponentially. Round to the nearest year.
- 21) Suppose that \$4000 is invested at an interest rate of 5.3% per year, compounded continuously. What is the balance after 3 years?
- 22) Suppose that \$9000 is invested at an interest rate of 5.4% per year, compounded continuously. What is the doubling time?

**Solve the problem.**

- 23) Ben Franklin bequeathed \$4000.00 to the city of Boston in 1790. Assuming the fund grew to \$6 million in 200 years,
- a) find the interest rate compounded continuously that would yield this total value.
  - b) How long did it take until the the balance reached \$1,000,000?
  - c) How fast was the balance changing when it reached \$1,000,000?

**Solve.**

- 24) A certain radioactive isotope has a half-life of approximately 1900 years. How many years to the nearest year would be required for a given amount of this isotope to decay to 80% of that amount?
- 25) An artifact is discovered at a certain site. If it has 57% of the carbon-14 it originally contained, what is the approximate age of the artifact to the nearest year? (carbon-14 decays at the rate of 0.0125% annually.)
- 26) A certain radioactive isotope decays at a rate of 0.3% annually. Determine the half-life of this isotope, to the nearest year.
- 27) The number of bacteria in a certain culture grows exponentially. If 5000 bacteria were present initially and 8,000 were present 10 minutes later, how long will it take for the number of bacteria to double?
- 28) The price of wheat per bushel at time  $t$  (in months) is  $f(t) = 4 + .001t + .01e^{-t}$ . What is the percentage rate of change at  $t = 1$ ?
- 29) A lake is stocked with 100 fish. After 3 months there are 250 fish. The formula for the number of fish in the lake  $t$  months after it is stocked is  $P(t) = \frac{1000}{1 + Be^{-1000kt}}$
- a) Find  $B$ .
  - b) Find  $k$
  - c) How many fish will be in the lake after 4 months?

**Solve the problem.**

30) The demand function for a certain product is given by

$$D(p) = 800e^{-0.1p},$$

where  $p$  is price per unit. Recall that total revenue is given by  $R(p) = pD(p)$ . At what price per unit  $p$  will the revenue be maximum?

31) The aerobic rating of a person  $x$  years old is modeled by the function:

$$A(x) = \frac{110(-2 + \ln x)}{x} \quad \text{for } x \geq 10$$

At what age is a person's aerobic rating largest?

**Graph**

32)  $y = 2xe^{-x}$

fn is increasing \_\_\_\_\_

fn is decreasing \_\_\_\_\_

fn has a relative max at \_\_\_\_\_

fn has a rel min at \_\_\_\_\_

fn is concave up \_\_\_\_\_

fn is concave down \_\_\_\_\_

fn has an inflection point at \_\_\_\_\_

# Answer Key

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- 1) a) 1                      b) 9                      c) 8/25                      d) 19/6
- 2) a)  $\ln \frac{1}{64} = -3$                       b)  $x = \ln(2/5)$
- 3) a)  $x = e^{1.6094}$                       b)  $e^{3.6636} = 39$
- 4)  $\ln(7)$
- 5)  $\ln x$
- 6)  $\ln \frac{17x(3)}{y^2}$
- 7)  $\ln \frac{w(w+5)^3}{(w-5)^6}$
- 8)  $\ln(y-12)^{2/3}(y+y)$
- 9) a) -1, -4                      b) -3, 1/2
- 10) a) 15.4033                      b)  $25 \ln 4$
- 11) a) 24,154,951.5                      b) 19/6
- 12) a)  $8xe^{x^2}$                       b)  $6xe^{3x^2} + 1$
- 13) a)  $5xe^{3x}(3x+2)$                       b)  $(x^2+1)e^x$
- 14) a)  $\frac{e^x(3x^2-6x+7)}{(3x^2+7)^2}$                       b)  $15x^2e^{x^3}(e^{x^3}-1)^4$
- 15) a)  $\frac{2x}{x^2+7}$                       b)  $\frac{1-5\ln x}{x^6}$
- 16) a)  $\frac{x e^x \ln x - e^x}{x \ln^2 x}$                       b)  $\frac{4e^{4x}}{e^{4x}-4}$
- 17) a)  $\frac{e^{8x}(1+8x \ln x)}{x}$                       b)  $y' = \frac{x}{x^2+1} - \frac{1}{2x+3}$
- 18)  $\frac{3t^2(t^6+1) + 6t^5(t^3-5)}{(t^3-5)(t^6+1)}$
- 19)  $y = 8x + 2$
- 20) 100 years
- 21) \$4689.35
- 22) 12.8 years
- 23) a) 3.7%                      b) about 149 years                      c) \$36,688.27/year
- 24) 612 years
- 25) 4497 years
- 26) 231 years
- 27) 14.75 years
- 28) -0.0667%
- 29) a)  $B = 9$                       b)  $k \approx .00037$                       c) about 328 fish
- 30) \$10
- 31) when  $x = 20.08$  or about 20 years old

Answer Key

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- 32) fn is increasing  $(-\infty, 1)$
- fn is decreasing  $(1, \infty)$
- fn has a relative max at  $(1, 2/e)$
- fn has a relative min at none
- fn is concave up  $(2, \infty)$
- fn is concave down  $(-\infty, 2)$
- fn has an inflection point at  $(2, 4/e^2)$

