

M118 SECTION 8.5 – RANDOM VARIABLE, PROBABILITY DISTRIBUTION, and EXPECTED VALUE

1) A random variable is a function that assigns a numerical value to each simple event in a sample space S.

Ex. Toss 3 coins. We are interested in the number of heads.

$$S = \{ TTT, TTH, THT, HTT, THH, HTH, HHT, HHH \}$$

Probability Distribution:

X	0	1	2	3
p(X)	1/8	3/8	3/8	1/8

Suppose we toss three coins a great number of times.

What would be the average number of heads per toss.

We would expect to get 0 heads 1/8th of the time.

We would expect to get 1 head 3/8th of the time.

We would expect to get 2 heads 3/8th of the time.

We would expect to get 3 heads 1/8th of the time.

So in order to calculate the expected value or expected average

1) Form the probability distribution of X

2) multiply each value of X by its probability - then sum it up

Expected Value $EV = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8) = 1.5$

Now we of course will never get 1.5 heads on any toss but the average over the long run is 1.5 heads.

#12

On three rolls of a single die, you will lose \$10 if a 5 turns up at least once, and you will win \$7 otherwise. What is the expected value of the game?

#28

Ten thousand raffle tickets are sold for \$2 each for a local library benefit. Prizes are awarded as follows: 2 prizes of \$1000, 4 prizes of \$500, and 10 prizes of \$100. What is the expected value of this raffle if you purchase 1 ticket?

#26

A game has an expected value to you of $-\$0.50$. It costs \$2 to play, but if you win you receive \$200. What is the probability of winning?

A game is fair if and only if $EV = 0$.

#16

A coin is tossed three times. Suppose you lose \$3 if 3 heads appear, lose \$2 if 2 heads appear, and win \$3 if 0 heads appear. How much should you win or lose if 1 head appears in order for the game to be fair.

#37

The annual premium for a \$5000 insurance policy against the theft of a painting is \$150. If the (empirical) probability that the painting will be stolen during the year is .01, what is your expected return from the insurance company if you take out this insurance?

#30

A box of 8 flashbulbs contain 3 defective bulbs. A random sample of 2 is selected and tested. Let X be the random variable associated with the number of defective bulbs in the sample.

a) Find the probability distribution of X .

b) Find the expected number of defective bulbs in a sample.