M118 SECTION 4.5 – INVERSE OF A SQUARE MATRIX

1) In the Real Numbers $1 \cdot a = a \cdot 1 = a$ 1 is the identity for multiplication.

Do the matrices have an identity matrix I such that IM = MI = M?

Identity matrices exist only for square matrices.

The identity Matrix for Multiplication (only for Square Matrices) is the square matrix of dimension $n \times n$, denoted by I, with 1's on the diagonal and 0's everywhere else.

$2\times2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $3\times3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $4\times4 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ etc.

Example: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$

$$IM = MI = M$$

In the real numbers there exists an inverse for the number a called a^{-1} such that

$$a^{-1} \cdot a = 1 \quad \Rightarrow \quad \frac{1}{2} \cdot 2 = 1$$

Do matrices have an inverse? Inverses of matrices do not always exist.

Ley M be a square matrix of dimension $n \times n$ and I be the Identity matrix. If there exists a matrix M^{-1} such that

$$M^{-1} \cdot M = M \cdot M^{-1} = I \text{ then } M^{-1} \text{ is called the inverse of } M$$

Let's find the inverse of $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$.

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$5a - 7c = 1$ $5b - 7d = 0$

$-2a + 3c = 0$ $-2b + 3d = 1$

Solving these systems of equations we find a = 3, b= 7, c= 2 and d= 5.

So the inverse of $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$ is $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$.

Sometimes a matrix inverse does not exist. (Ex. 4)

$$\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \text{ has no inverse}$$

Introduce Inverse on Calculator

Matrices that do not have an inverse are called singular.

APPLICATION – CRYPTOGRAPHY

A	B	C	D	E	F	...	X	Y	Z	blank
1	2	3	4	5	6		24	25	26	0 or 27

Encode I N D I A N A _ U N I V E R S I T Y
9, 14, 4, 9, 1, 14, 1, 27, 21, 14, 9,22, 5, 18,19, 9, 20, 25

For Example Use $\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix} = A$

$$B = \begin{bmatrix} 9 & 4 & 1 & 1 & 21 & 9 & 5 & 19 & 20 \\ 14 & 9 & 14 & 27 & 14 & 22 & 18 & 9 & 25 \end{bmatrix}$$

$$AB = \begin{bmatrix} 78 & 43 & 46 & 85 & 126 & 102 & 74 & 103 & 155 \\ 23 & 13 & 15 & 28 & 35 & 31 & 23 & 28 & 45 \end{bmatrix}$$

Example: The following message was encoded with matrix A. Decode this message.

$$A = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix}$$

9 13 40 49 29 34 2 3 22 26 6 9 43 57 29 34 54 74