M118 SECTION 4.4– BASIC OPERATIONS OF MATRICES

1) Two matrices are equal if they have the same size and the corresponding elements are equal.

1) **Addition of Matrices is only defined for matrices of the same size:** The sum of two matrices is the matrix with elemnts that are the sum of the corresponding elements.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

A **zero matrix** is one where all elements are zeroes.

The **negative of a matrix M**, denoted by –M, is the matrix with elements that are negatives of the elements of M:

$$\text{If } M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{then } -M = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

2) **Subtraction is defined A–B = A + (–B)**

$$\begin{bmatrix} 3 & -2 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ 2 & -4 \end{bmatrix}$$

3) **Scalar Multiplication**: Product of a number k and a matrix M, kM, is formed by multiplying each element of M by k

$$\text{so} \quad 3\begin{bmatrix} 4 & -1 & 0 \\ 2 & 5 & 6 \\ -3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -3 & 0 \\ 6 & 15 & 18 \\ -9 & 0 & 12 \end{bmatrix}$$

4) **Matrix Product**:

The product of a $1 \times n$ matrix and an $n \times 1$ matrix

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} ad + be + cf \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 2(-5)+(-3)(4)+1(-3) \end{bmatrix} = \begin{bmatrix} -10-12-3 \end{bmatrix} = \begin{bmatrix} -25 \end{bmatrix}$$

In order to multiply two matrices, AB, the number of columns of matrix A has to be the same as the number of rows of matrix B.

If matrix A has dimension m×p and matrix B has dimension p×q, then the product AB is defined but BA is not.

a) $\begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 2 & 0 \end{bmatrix}$ This product is not defined.

b) $\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

} This shows AB ≠ BA

d) $\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -12 \\ 3 & 6 \end{bmatrix}$