M118 SECTION 4.2– MATRICES

1) A **MATRIX** is a rectangular array of numbers written within brackets.

$$\mathbf{A} = \begin{bmatrix} 1 & -4 & 5 \\ 2 & 0 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 6 & 5 \\ 2 & 4 \\ 0 & 4 \\ -7 & 9 \end{bmatrix}$$

Each number in a matrix as called an **element**.

Matrix A has 2 rows and 3 columns

Matrix B has 4 rows and 2 columns

Dimension of a matrix is (the # of rows $\times$ the # of columns)

So, Matrix A has dimension 2×3 and Matrix B has dimension 4×2.

$$1 \times 4 \sim \begin{bmatrix} 2 & 3 & 4 & 5 \end{bmatrix} \qquad 4 \times 1 \sim \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix}$$

The position of an element is given by the row and column of the element.

In Matrix A $a_{1,1} = 1$ $a_{1,2} = -4$ $a_{2,3} = 3$

A **square matrix** has the same number of rows as columns $n \times n$.

The **principal diagonal** are the elements $a_{1,1}$, $a_{2,2}$, $a_{3,3}$ . . .

In a system of linear equations:

$$\begin{aligned} 2x - 3y &= 5 \\ 4x + 6y &= -2 \end{aligned}$$

Coefficient Matrix **Constant Matrix** **Augmented Matrix**

$$\begin{bmatrix} \mathbf{2} & \mathbf{-3} \\ \mathbf{4} & \mathbf{6} \end{bmatrix} \qquad \begin{bmatrix} \mathbf{5} \\ \mathbf{-2} \end{bmatrix} \qquad \left[\begin{array}{cc|c} \mathbf{2} & \mathbf{-3} & \mathbf{5} \\ \mathbf{4} & \mathbf{6} & \mathbf{-2} \end{array}\right]$$

From Section 4.1 we learned that two systems are equivalent if

1) Two equations are interchanged
2) One equation is multiplied by a nonzero constant
3) A constant multiple of one is added to another.

So, $2x - y = -7$

$x + 2y = 4 \quad \approx \left[\begin{array}{cc|c} 2 & -1 & -7 \\ 1 & 2 & 4 \end{array}\right]$ we want to get to $\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right] \Rightarrow x = m, y = n$

$$\left[\begin{array}{cc|c} 2 & -1 & -7 \\ 1 & 2 & 4 \end{array}\right] \underset{R_1 \to R_2}{\approx} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & -1 & -7 \end{array}\right] \underset{-2R_1 + R_2 \to R_2}{\approx} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -5 & -15 \end{array}\right] \underset{R_2/5 \to R_2}{\approx} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 3 \end{array}\right] \underset{2R_2 + R_1 \to R_1}{\approx} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right] \Rightarrow x = -2 \quad y = 3$$

Example: $-2x + 6y = 6$
$3x - 9y = -9$

$$\left[\begin{array}{cc|c} -2 & 6 & 6 \\ 3 & -9 & -9 \end{array}\right] \approx \left[\begin{array}{cc|c} 1 & -3 & -3 \\ 1 & -3 & -3 \end{array}\right] \approx \left[\begin{array}{cc|c} 1 & -3 & -3 \\ 0 & 0 & 0 \end{array}\right]$$ Infinite number of solutions

$x - 3y = -3$ so let $x = t$ then $t - 3y = -3$

$$-3y = -t - 3$$

$$y = \frac{-t-3}{-3} = \frac{t+3}{3} \qquad \left(t, \frac{t+3}{3}\right)$$

Example: $2x - y = 3$

$4x - 2y = -1 \quad \approx \left[\begin{array}{cc|c} 2 & -1 & 3 \\ 4 & -2 & -2 \end{array}\right] \approx \left[\begin{array}{cc|c} 1 & -1/2 & 3/2 \\ 0 & 0 & -7 \end{array}\right] \approx$ no solution

SUMMARY:

FORM 1	FORM 2	FORM 3
$\left[\begin{array}{cc\|c} 1 & 0 & m \\ 0 & 1 & n \end{array}\right]$	$\left[\begin{array}{cc\|c} 1 & m & n \\ 0 & 0 & 0 \end{array}\right]$	$\left[\begin{array}{cc\|c} 1 & m & n \\ 0 & 0 & p \end{array}\right]$
$x = m \quad y = n$	Infinite number of solutions	No solution