

## M118 SECTION 11.5 – NORMAL DISTRIBUTION

- 1) We can obtain a bell-shaped curve by joining the midpoints of a histogram with a smooth curve. These bell-shaped curves are called normal curves or normal probability distributions. They are completely determined by mean,  $\mu$ , and standard deviation,  $\sigma$ .

Properties of Normal Curves:

- 1) Bell-shaped – symmetrical with respect to vertical line thru the mean
- 2) Mean is a point where the axis of symmetry intersects the horizontal axis.
- 3) Shape is determined by mean and standard deviation
  - a) small  $\sigma$  = tight clustering about the mean  
tall, narrow curve
  - b) large  $\sigma$  = large deviation from mean  
broad flat curve
- 4) No matter the shape – area between the curve and x-axis is 1.
- 5) No matter the shape –
  - 68.3% of area lies within 1 standard deviation from the mean
  - 95.4% of area lies within 2 standard deviations from the mean
  - 99.7% of area lies within 3 standard deviations from the mean

Until now we have dealt with discrete random variables (finite)

Now we will have continuous random variables. (weights, light bulb lifetime, length of waiting time, etc.)

### AREAS UNDER NORMAL CURVES:

In order to find area under a normal curve, we can use the same table for all. The area under a normal curve between a mean,  $\mu$ , and a given number of standard deviations to the right or left is the same regardless of shape.

Thus let  $z$  represent the number of standard deviations that a measurement  $x$  is from a mean  $\mu$ .

$$\text{So, } z = \frac{x - \mu}{\sigma}$$

**Example:** We have a normal distribution with mean of 500 and standard deviation of 100.

a) What percentage lies between 500 – 670 hours?

b) What percentage lies between 500 and 750 hours?

c) What percentage lies between 385 – 500 hours?

d) What percentage lies between 400 – 560 hours?

**PROPERTIES OF NORMAL PROBABILITY DISTRIBUTION:**

**$P(a \leq x \leq b)$  = area under normal curve from a to b**

**$P(-\infty \leq x \leq \infty) = 1$  Total area under the curve**

**$P(x = c) = 0$  (area would be 0)**

Example: A normal random variable  $X$  has mean,  $\mu = 40$  and standard deviation,  $\sigma = 16$ .

a) Find the area under the curve between 16 and 60.

b) Find the area under the curve where  $x \geq 68$ .

#54

The average lifetime for a certain brand of car battery is 170 weeks, with a standard deviation of 10 weeks. If the company guarantees the battery for 3 years, what percentage of the batteries sold would be expected to be returned before the end of the warranty period? Assume a normal distribution.

#56

An automated manufacturing process produces a component with an average width of 7.55 cm, with a standard deviation of 0.02 cm. All components deviating by more than 0.05 cm from the mean must be rejected. What percentage of parts must be rejected, on the average? Assume a normal distribution.