

M118 SECTION 11.4 – Bernoulli Trials and Binomial Distribution

- 1) Bernoulli Experiment - 1) An experiment for which there are only 2 possible outcomes (flip a coin, win or lose a ball game, or pull a red or white ball out of an urn
- 2) We refer to one of the 2 outcomes as a success (S) and the other as a failure (F).
Let $\text{Prob}(S) = p$ and $\text{Prob}(F) = q$
so $1-p = q$
- 3) all trials are independent

Find p and q for a single roll of a fair die, where a success is a number divisible by 3 turning up.

$$p = \text{Prob}(\# \text{ divisible by } 3) = 2/6 = 1/3$$

$$q = \text{Prob}(\# \text{ NOT divisible by } 3) = 2/3 \text{ or } 1 - 1/3 = 2/3$$

Suppose we roll a die 4 times. We want to know the probability of obtaining exactly 2 threes.

Use a tree

$$\text{Prob}(3,3,3',3') =$$

$$\text{prob}(3,3',3,3') =$$

$$\text{prob}(3,3',3',3) =$$

$${}^4C_2 = \frac{4!}{2!2!} = 6$$

$$\text{prob}(3',3,3,3') =$$

$$\text{prob}(3',3,3',3) =$$

$$\text{prob}(3',3',3,3) =$$

$$\text{prob}(\text{exactly } 2 \text{ 3's}) = {}^4C_2 (1/6)^2 (5/6)^2 =$$

THEOREM 1: The probability of exactly x successes in n independent Bernoulli trials, where $\text{prob}(S) = p$ and $\text{prob}(F) = 1-p = q$

$$P(\text{exactly } x \text{ successes in } n \text{ trials}) = {}_n C_x p^x q^{n-x}$$

If a fair die is rolled 5 times, what is the probability of rolling:

a) exactly 2 - 3's

b) at least 2- 3's

c) exactly one 3

d) at least one 3

#28

If 60% of the electorate supports the mayor, what is the probability that in a random sample of 10 voters, fewer than half support her?

MEAN AND STANDARD DEVIATION OF BERNOULLI EXPERIMENT:

$$\text{mean} = \mu = np$$

$$\text{std dev} = \sigma = \sqrt{npq}$$

#30 Construct a histogram for the binomial distribution. Also compute the mean and standard deviation.

$$P(x) = {}_6C_x (.6)^x (.4)^{(6-x)}$$

#58

The probability that a marriage will end in divorce within 10 years is .4. What are the mean and standard deviation for the binomial distribution involving 1,000 marriages?

#44

If the probability of a new employee in a fast food chain still being with the company at the end of 1 year is .6, what is the probability that out of 8 newly hired people

a) 5 will still be with the company after 1 year?

b) 5 or more will still be with the company after 1 year?

#46

A manufacturing process produces, on the average, 3% defective items. The company ships 10 items in each box and wishes to guarantee no more than 1 defective item per box. If this guarantee accompanies each box, what is the probability that the box will fail to satisfy the guarantee?

#50

A drug company claims that a drug produces serious side effects in 20 people out of 1,000, on the average. To check this claim, a hospital administers the drug to 10 randomly chosen patients and finds that 3 suffer serious side effects. If the drug company claims are correct, what is the probability of the hospital obtaining these results?