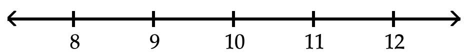
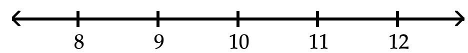
M118 SECTION 11.3 – Measures of Dispersion

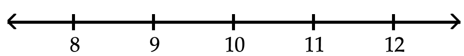
1) 1 – RANGE

UNGROUPED – Largest Data Measurement – Smallest Data Measurement

GROUPED – Upper Boundary – Lowest Boundary







2) **STANDARD DEVIATION**:

UNGROUPED DATA

Suppose we have 5 measurements 5.2, 5.3, 5.2, 5.5, 5.3

$\bar{x} \approx 5.3$

How much variation exists between the mean and each measurement in the sample.

Let deviation = $x_i - \bar{x}$

x_i	$x_i - \bar{x}$
5.2	−.1
5.3	0
5.2	−.1
5.5	.2
5.3	0

Take the mean of the deviation = $\frac{-.1 + 0 + -.1 + .2 + 0}{5} = 0$

The deviation will always be 0 so that doesn't tell us anything, so let's take each deviation square it and then find the mean of those numbers (This is called the variance)

$$\sigma^2 = \text{variance} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^{5} (x_i - 5.3)^2}{5} \approx .012 \text{ sq. cm}$$

Now the variance is in square cm and the original numbers are in cm – so in order to get the same units we will take the square root of the variance and we call that the

$$\sigma = \textbf{standard deviation} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum_{i=1}^{5} (x_i - 5.3)^2}{5}} \approx .11 \text{ cm}$$

The variance σ^2 and standard deviation σ are the population variance and standard deviation. It has been shown that the sample variance and sample standard deviation is a better estimate if we divide by n–1 instead of n.

So, in this class, we will use the

sample variance = $s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$ and

sample standard deviation = $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$

So, for our example $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - 5.3)^2}{5 - 1}} \approx .12 \text{ cm}$

Match Problem 1: Find the standard deviation for the sample measurements: 1.2, 1.4, 1.7, Match Problem 1: Find the standard deviation for the sample measurements: 1.2, 1.4, 1.7, 1.3, 1.5

Do on calculator:

STANDARD DEVIATION FOR GROUPED DATA:

n = # of measurements
k = # of classes
x_i = midpoint of the ith class
f_i = frequency of the ith class

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2 f_i}{n - 1}}$$

Find the sample standard deviation for:

	midpoint	frequency
7.5 – 8.5	8	1
8.5 – 9.5	9	2
9.5 – 10.5	10	4
10.5 – 11.5	11	2
11.5 – 12.5	12	1

$\bar{x} = 10$

$$s = \sqrt{\frac{(8-10)^2(1) + (9-10)^2(2) + (10-10)^2(4) + (11-10)^2(2) + (12-10)^2(!)}{10-1}}$$

≈ 1.15

STANDARD DEVIATION ON CALCULATOR:

Midpoints in L_1

Frequency in L_2 2nd List → Math → 7:stdDev(

#6 Find the standard deviation for the following data:

Interval	Frequency
0.5 – 3.5	5
3.5 – 6.5	1
6.5 – 9.5	2
9.5 – 12.5	7

In a bell shaped curve:

About 68% of the data lies within 1 standard deviation of the mean

($\bar{x}$ – s, $\bar{x}$ + s)

About 95% of the data lies within 2 standard deviations of the mean

($\bar{x}$ – 2s, $\bar{x}$ + 2s)

Almost 100% of the data lies within 3 standard deviations of the mean

($\bar{x}$ – 3s, $\bar{x}$ + 3s)

#4 What proportion of the following sample of ten measurements lies

a) within 1 standard deviation of the mean? ___________________

b) within 2 standard deviations of the mean? ___________________

c)within 3 standard deviations of the mean? ___________________

3 5 1 2 1 5 4 5 1 3

Based on your answers above, would you conjecture that the histogram is approximately bell shaped?

What proportion of the following sample of measurements lies
a) within 1 standard deviation of the mean? ___________________

b) within 2 standard deviations of the mean? ___________________

c)within 3 standard deviations of the mean? ___________________

2	8	0	9	2	8	1
3	5	6	0	1	9	4
5	8	2	1	3	7	9

$\bar{x} = 4.4$ $s = 3.2$