

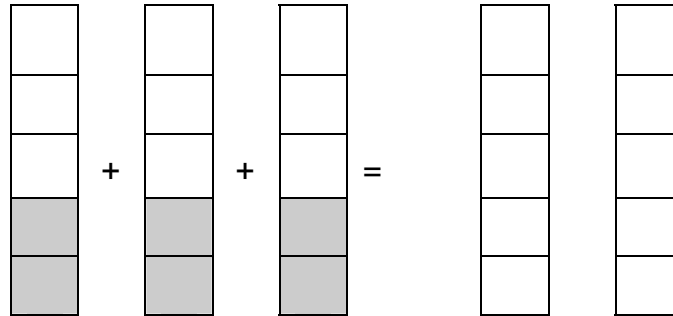
5.3 Multiplication and Division of Rational Numbers

I. MULTIPLICATION

A. Models for Multiplication

1. Repeated Addition $3 \times \frac{2}{5}$

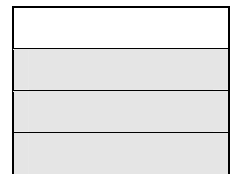
2. Area Model $3 \times \frac{2}{5}$



B. But, what happens when both multiplicands are fractions? For example, three-fourths of the students at the local elementary school buy their lunch. Two-fifths of these students are girls. What part of the student body are girl lunch buyers?

C. Let's find "two-fifths" of "three-fourths" that is, $\frac{2}{5} \times \frac{3}{4}$

1. Divide the area into 4ths. Shade 3. (This is $\frac{3}{4}$'s of the student body.)
2. Divide the whole area in two fifths (opposite direction)
3. Count the overlap of shading.



Use area models to solve the following problems:

$$\frac{2}{3} \text{ of } \frac{1}{4}$$

$$\frac{1}{3} \times \frac{5}{6}$$

DEFINITION OF MULTIPLICATION OF RATIONAL NUMBERS**(THE ALGORITHM)**

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then $\frac{a}{b} \times \frac{c}{d} =$

$$\frac{5}{6} \times \frac{3}{4}$$

$$\frac{12}{25} \cdot -\frac{5}{6}$$

$$-8 \cdot \frac{3}{7}$$

Let's try the next problem using two different methods:

$$3\frac{1}{2} \times 7\frac{2}{3} \quad (\text{change to improper fractions})$$

$$3\frac{1}{2} \times 7\frac{2}{3} \quad (\text{Use FOIL})$$

#16 Di Paloma University had a faculty reduction and lost $\frac{1}{5}$ of its faculty. If 320 faculty members were left after the reduction, how many were there originally?

On Wednesday it rained $1\frac{1}{2}$ inches and on Thursday it rained only $\frac{1}{3}$ as much as it had on Wednesday. How much did it rain on Thursday?

PROPERTIES OF RATIONAL NUMBER MULTIPLICATION

Given any rational numbers, $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$

1. Closure

2. Commutative

3. Associative

4. Multiplicative Identity of Rational Numbers

The number _____ is the unique number such that for every rational number $\frac{a}{b}$, $\frac{a}{b} \cdot 1 = 1 \cdot \frac{a}{b} = \frac{a}{b}$

5. Multiplicative Inverse of Rational Numbers

For any nonzero rational number $\frac{a}{b}$, $\frac{b}{a}$ is the unique rational number such that $\frac{a}{b} \cdot \frac{b}{a} = 1 = \frac{b}{a} \cdot \frac{a}{b}$

(The multiplicative inverse of $\frac{a}{b}$ is also called the *reciprocal* of $\frac{a}{b}$.)

6. Distributive Property of Multiplication over Addition for Rational Numbers

If $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ are any rational numbers, then $\frac{a}{b} \left(\frac{c}{d} + \frac{e}{f} \right) =$

7. Multiplication Property of Equality for Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers such that $\frac{a}{b} = \frac{c}{d}$, and $\frac{e}{f}$ is any rational number, then

$$\frac{a}{b} \cdot \frac{e}{f} = \frac{c}{d} \cdot \frac{e}{f}$$

8. Multiplication Property of Zero for Rational Numbers

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b} \cdot 0 = 0 \cdot \frac{a}{b} = 0$

Find the multiplicative inverse for the following:

$\frac{4}{5}$

0

$-\frac{3}{4}$

7

$3\frac{1}{5}$

Now find the additive inverse of each of the rational numbers above.

Solve the following equations:

$$-\frac{3}{7}x = 6$$

$$\frac{2}{5}x = \frac{14}{15}$$

Five-eighths of the students at IUS live in dormitories. If 6000 students live in the college dormitories, how many students are there in the college?

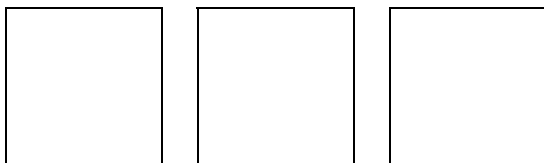
II. DIVISION

AREA MODELS

Recall that $18 \div 6$ means "how many 6's are there in 18?"

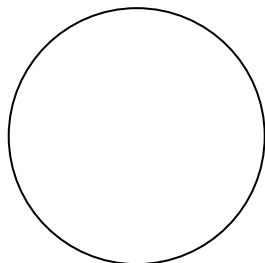
So, $3 \div \frac{1}{2}$ means "**how many halves are there in 3**".

Here are 3 "wholes". How many "halves" are there here? _____



$$\left(\frac{3}{4}\right) \div \left(\frac{1}{8}\right)$$

This means how many _____ are there in _____.



DEFINITION OF DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers and $\frac{c}{d}$ is not zero, then $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$ if, and only if, $\frac{e}{f}$ is the unique rational number such that $\frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b}$.

EXAMPLES: Interpret the following as a “How many ___ are there in ___” and solve.

$$2 \div \frac{1}{5}$$

Algorithm for Division of Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \text{ where } \frac{c}{d} \neq 0$$

$$\frac{9}{14} \div -\frac{2}{3}$$

$$2\frac{3}{4} \div \frac{5}{6}$$

We have $35\frac{1}{2}$ yards of material available to make shirts. Each shirt requires $\frac{3}{8}$ yard of material. How many shirts can we make? How much material will be left over?

III. MENTAL MATHEMATICS AND ESTIMATION

A. MENTAL MATHEMATICS

(Use mental mathematics to find exact answer. Describe in a sentence. You are looking for a way beyond the algorithm – use the properties and/or what you know about mixed numbers.)

$$(36 \cdot 25) \cdot \frac{1}{9}$$

$$5\frac{1}{6} \cdot 12$$

$$6\frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{4}{5} \cdot 20$$

B. ESTIMATION

(Get an answer "in the ballpark". Describe in a sentence.)

$$3 \frac{1}{4} \cdot 7 \frac{8}{9}$$

$$24 \frac{6}{7} \div 4 \frac{3}{8}$$

IV. PROPERTIES OF EXPONENTS

RULE	EXAMPLE
a^m means	4^3
$a^m \cdot a^n$	$x^3 \cdot x^5$
$(a^m)^n$	$(y^3)^2$
$(ab)^m$	$(2xy^2)^3$
$\frac{a^m}{a^n}$	$\frac{x^5}{x^3}$
$\left(\frac{a}{b}\right)^m$	$\left(\frac{x^3}{2y}\right)^5$
$\frac{a^n}{a^n}$	$\frac{k^7}{k^7}$
a^{-m}	2^{-3} x^{-5}
$\left(\frac{a}{b}\right)^{-m}$	$\left(\frac{1}{2}\right)^{-4}$ $\left(\frac{3x}{y}\right)^{-2}$

EXAMPLES**(LOOK FOR SHORTCUTS!!)**

$$\left(\frac{2}{3}\right)^3 \cdot \left(\frac{4}{9}\right)^6$$

$$\left(\frac{1}{2}\right)^9 \div \left(\frac{1}{2}\right)^6$$

$$\left(\frac{a^2}{b}\right)^7$$

SOLVE THE FOLLOWING EQUATIONS

$$n^2 = 36$$

$$9^x = 27$$

$$3^n = 9^2$$

$$2^n \cdot 2^7 = 8$$

$$3^n = 27^5$$