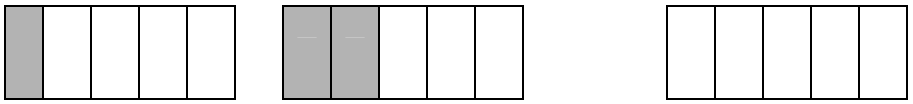
T102 SECTION 5.2 THE SET OF RATIONAL NUMBERS

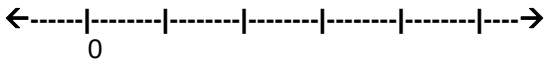
I. ADDITION WITH "LIKE" DENOMINATORS

DEFINITION OF ADDITION OF RATIONAL NUMBERS (like denominators)

If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$



$\frac{1}{5}$ + $\frac{2}{5}$ =

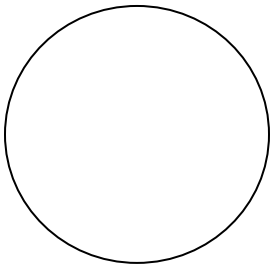


So, $\frac{1}{5} + \frac{2}{5} =$

II. ADDITION WITH "UNLIKE" DENOMINATORS

But what if the fractions do not have the same denominator?

For instance, $\frac{1}{3} + \frac{1}{4}$

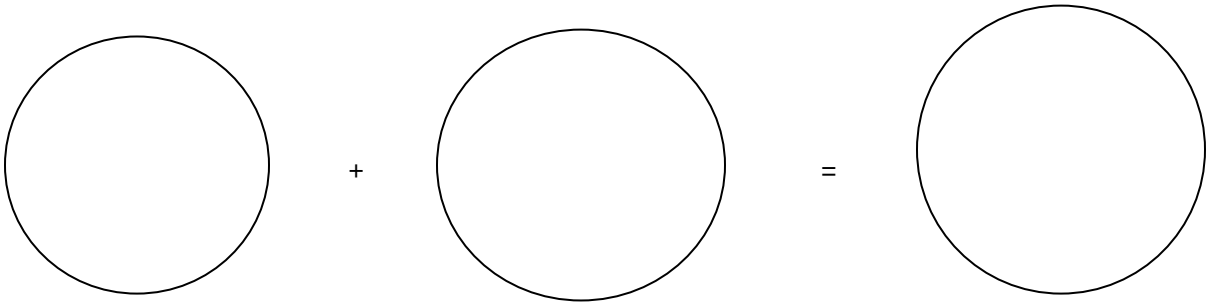


But, how do we count this? We need to find a way to combine the two drawings to find the sum. Let's "build-up" each fraction:

$\frac{1}{3} = \frac{\ }{6} = \frac{\ }{9} = \frac{\ }{12} = \frac{\ }{15}$

$\frac{1}{4} = \frac{\ }{8} = \frac{\ }{12} = \frac{\ }{16} = \frac{\ }{20}$

Now, compare these two lists and look for a "like" denominator. What is it? _____



ADDITION OF RATIONAL NUMBERS (Unlike denominators)

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

EXAMPLES: Using the LCD Using the Above Property

$\frac{3}{16}+\frac{2}{5}$

$\frac{5}{12}+\frac{-1}{8}$

$\frac{5}{x}+\frac{-3}{y}$

$\frac{5}{24}+\frac{11}{26}$

$\frac{5}{48}+\frac{7}{36}+\frac{1}{54}$

Application

A clerk sold three pieces of one type of ribbon to different customers. One piece was $\frac{1}{3}$ yd long, another was $2\frac{3}{4}$ yd long, and the third was $1\frac{7}{8}$ yd long. What was the total length of that type of ribbon sold?

III. MIXED NUMBERS

Mixed numbers are numbers that are the **sum** of an integer and a fractional part of an integer. For example, if a nail is $2\frac{3}{4}$ inches long, this means 2 inches **plus an additional** $\frac{3}{4}$ inches. (It is common to think that since *2y* means *2* times *y,* that $2\frac{3}{4}$ means 2 times $\frac{3}{4}$, but this is ***incorrect!***)

Change the following ***mixed numbers to improper fractions***.

Using the Conventional Algorithm	Change with Meaning
$2\frac{3}{4}$	$2\frac{3}{4} = 2+\frac{3}{4} =$
$-5\frac{3}{7}$	$-5\frac{3}{7}$

Change the following ***improper fractions to mixed numbers***.

Using the Conventional Algorithm	Change with Meaning
$\frac{13}{4}$	$\frac{13}{4} = \frac{4+4+4+1}{4} =$
$\frac{-29}{6}$	$\frac{-29}{6}$

PROPERTIES OF ADDITION FOR RATIONAL NUMBERS

Given any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ where, b and d are non-zero integers:

1. Closure
2. Commutative
3. Associative
4. Additive Identity
5. Additive Inverse — For any rational number $\frac{a}{b}$ there exists a unique number _____ such that:

Name the additive inverse of the following:

$\frac{5}{6}$ $\quad -\frac{2}{5}$ $\quad -\left(-5\frac{3}{7}\right)$

V. ADDITION OF MIXED NUMBERS (Know how to add using the given mixed numbers)

*** Using improper fractions for addition of mixed numbers is extremely inefficient and can result in errors, especially if the whole number is large.*

$2\frac{3}{8}+5\frac{5}{6}$

$$\begin{array}{r} 2\frac{3}{8} \\ +5\frac{5}{6} \\ \hline \end{array}$$

$82\frac{9}{25}+17\frac{32}{50}$

VI. SUBTRACTION OF RATIONAL NUMBERS

SUBTRACTION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

SUBTRACTION IF Unlike Denominators

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$\frac{3}{4} - \frac{1}{3} =$

$\frac{3}{7} - \frac{5}{6} =$

SUBTRACTION OF MIXED NUMBERS (Know how to subtract using the given mixed numbers)

$7\frac{1}{3}-4\frac{4}{5}$

$$\begin{array}{r} 5\frac{1}{3} \\ -\ 2\frac{3}{4} \\ \hline \end{array}$$

$12\frac{2}{9}+-15\frac{5}{6}$

$5\frac{7}{12} - 2$ Versus $8 - 2\frac{3}{4}$ $345\frac{1}{8} - 276\frac{2}{3}$

Martine bought $8\frac{3}{4}$ yd of fabric. She wants to make a skirt using $1\frac{7}{8}$ yd, pants using $2\frac{3}{8}$ yd, and a vest using $1\frac{2}{3}$ yd. How much fabric will be left over?

VII. ESTIMATION WITH RATIONAL NUMBERS

Many times when estimating with fractions, it is helpful to round to a *convenient fraction* – for instance: $0,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{2}{3},\ \frac{3}{4},\ \text{or}\ 1$.

For example, if you got 59 out of 80 questions correct on your test, this is about $\frac{60}{80}$ or $\frac{6}{8}$ or $\frac{3}{4}$

Then we can conclude that $\frac{3}{4}$ is a HIGH ESTIMATE.

(You actually got less than $\frac{3}{4}$ of the test correct, since 59 < 60, then $\frac{59}{80} < \frac{60}{80} = \frac{3}{4}$).

Approximate each of the following using $0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{4},$ or 1. Tell if your estimate is low or high.

$\frac{3}{197}$ $\frac{8}{9}$ $\frac{47}{64}$ $\frac{2002}{2000}$

$\frac{62}{31} + \frac{99}{98}$

Explain the error in each of the following:

a) $\frac{13}{35} = \frac{1}{5};\ \frac{27}{73} = \frac{2}{3};\ \frac{16}{64} = \frac{1}{4}$

b) $\frac{4}{5} + \frac{2}{3} = \frac{6}{8};\ \frac{2}{5} + \frac{3}{4} = \frac{5}{9};\ \frac{7}{8} + \frac{1}{3} = \frac{8}{11}$

c) $8\frac{3}{8} - 6\frac{1}{4} = 2\frac{2}{4};\ 5\frac{3}{8} - 2\frac{2}{3} = 3\frac{1}{5};\ 2\frac{2}{7} - 1\frac{1}{3} = 1\frac{1}{4}$