SECTION 5.1 THE SET OF RATIONAL NUMBERS

I. INTRODUCTION

Students life experiences include fractions, $\frac{1}{5}$ or a pie; three kids share two oranges, etc.

The set of ***rational numbers***, denoted **Q**, is the set of all numbers of the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$. The number *a* is called the ____________________ and the number *b* is called the ______________________.

We can write this using set builder notation as: **Q** = $\left\{ \frac{\quad}{\quad} \qquad\qquad\qquad\qquad\qquad\qquad \right\}$

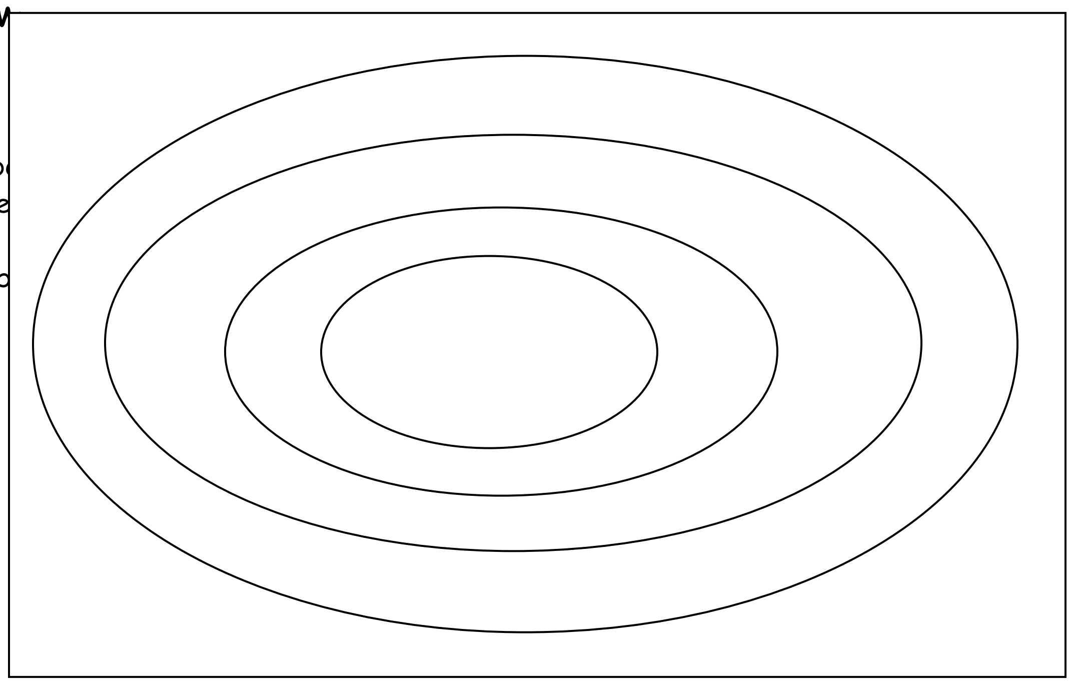
A ***fraction*** is any number of the form $\frac{a}{b}$ where *a* and *b* are *any numbers (not necessarily integers)*, with
$b \neq 0$. (Note that all rational numbers are fractions, but not all fractions are rational numbers.)

Examples: Classify as fraction, rational number or both.

$\frac{-4}{7}$ $\frac{-1}{\sqrt{2}}$ $\frac{0}{7}$ -5

VENN DIAGRAM OF THE REAL NUMBERS

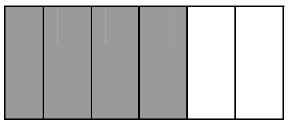
N: Natural Numbers
W: Whole Numbers
I: Integers
Q: Rational Numbers
R: Real Numbers

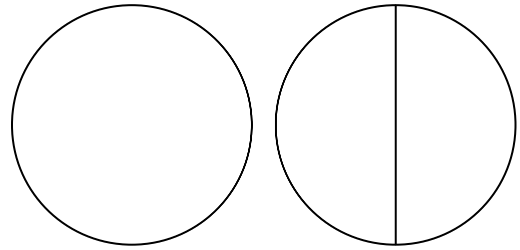


TRUE OR FALSE: $\mathbf{I} \subseteq \mathbf{Q}$ $\mathbf{W} \subseteq \mathbf{I}$ $\mathbf{I} \subseteq \mathbf{N}$ $\mathbf{N} \subseteq \mathbf{W}$ $\mathbf{N} \subseteq \mathbf{Q}$ $\mathbf{W} \subseteq \mathbf{Q}$

II. MODELS FOR FRACTIONS

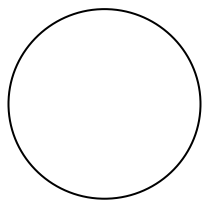
A. COLORED REGION (or AREA) MODEL



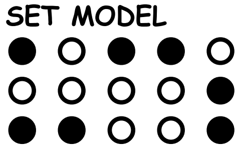


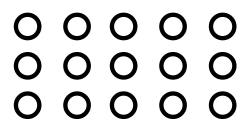
What fraction is shaded? __________ What fraction is shaded? __________

Shade $\frac{3}{4}$.



B. SET MODEL





What fraction of the balls are shaded? Shade ⅓ of the balls.

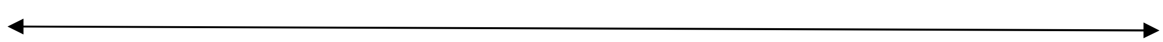
C. NUMBER LINE MODEL

Shade 4/5 of the unit segment |--------|--------|--------|--------|--------|

0 1

Place the following rational numbers on a number line at the proper intervals:

$$2\ ,\quad \frac{2}{3},\quad -\frac{3}{2}\ ,\quad 0\ ,\quad -\frac{3}{4},\quad -1\frac{7}{8}\ ,\quad 1\ ,\quad \frac{7}{5},\quad -2$$



D. FRACTION STRIP MODEL (See Appendix A-29 in the Activities Book)

Find strips that can be folded into parts so that the resulting strip is equal in length to the given fraction. Folds may be made ONLY on the lines of the strips.

$\frac{1}{2} = \qquad = \qquad = \qquad =$

$\frac{6}{8} =$

Proper Fractions

A fraction $\frac{a}{b}$ where $0 \leq |a| < |b|$

Simply put, a proper fraction represents a number greater than _______ and less than _________

Examples:

Improper Fractions

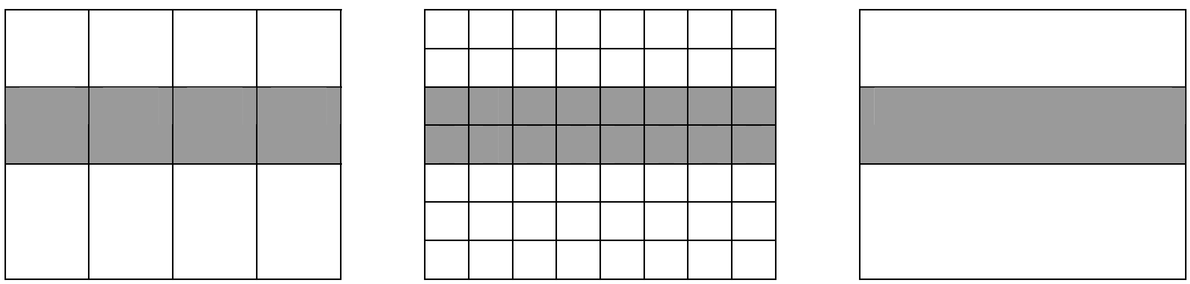
A fraction $\frac{a}{b}$ where $|a| \geq |b| > 0$

Simply put, an improper fraction represents a number greater than or equal to __________ and less than or equal to _________

Examples:

Fraction Activity with Fraction Circles *(T101 – Fractions Activity)*

III. EQUIVALENT OR EQUAL FRACTIONS



$\frac{\quad}{12} = \frac{\quad}{48} = \frac{\quad}{3}$

Notice that the *value* of the fraction does not change if its numerator and denominator are multiplied (or divided) by the same nonzero whole number.

FUNDAMENTAL LAW OF FRACTIONS

Let $\frac{a}{b}$ be any fraction and *n* a nonzero whole number, then $\frac{a}{b} = \frac{a \cdot n}{b \cdot n}$

We use this law to "build up" fractions (ie. common denominators) and also to simplify (ie. reduce) them.

Find the value for *x* such that : $\frac{12}{15} = \frac{x}{45}$ $\frac{12}{x} = \frac{1}{5}$

IV. SIMPLIFYING FRACTIONS

DEFINTION OF SIMPLEST FORM

A rational number $\frac{a}{b}$ is in simplest form if *a* and *b* have no common factor greater than 1, that is, *a* and *b* are relatively prime (that is, the GCD (*a*, *b*) = 1).

Express in Simplest Form

$\frac{28}{56}$ $\frac{-27}{32}$ $\frac{28ab^2}{42a^2b^2}$

$\frac{(a+b)^2}{3a+3b}$ $\frac{x^2+x}{x+1}$ $-\frac{7x}{15y}$

$\frac{3+x^2}{3x^2}$ $\frac{3+3x^2}{3x^2}$ $\frac{a^2-b^2}{a+b}$

V. EQUALITY OF FRACTIONS

We can use three methods to determine if two fractions are equal.

A. Simplify (reduce) both fractions

$\frac{12}{42}$ $\frac{10}{35}$

B. Rewrite both fractions using the *Least Common Denominator* (the LCM of the denominators)

$\frac{15}{24}$

$\frac{10}{16}$

C. Rewrite both fractions with *any common denominator*

$\frac{27}{72}$

$\frac{15}{40}$

This last method leads us to the following property:

PROPERTY Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal if, and only if, *ad = bc.*

Use the above property to determine if the following fractions are equal:

$\frac{14}{63}$ and $\frac{10}{45}$ $\frac{17}{27}$ and $\frac{25}{45}$

VI. ORDERING FRACTIONS

For some students comparisons are instinctive, based upon life experiences.

A. Like Denominators: It is simple to compare fractions that have like denominators; just compare the numerators. The one with the greater numerator is the greater fraction.

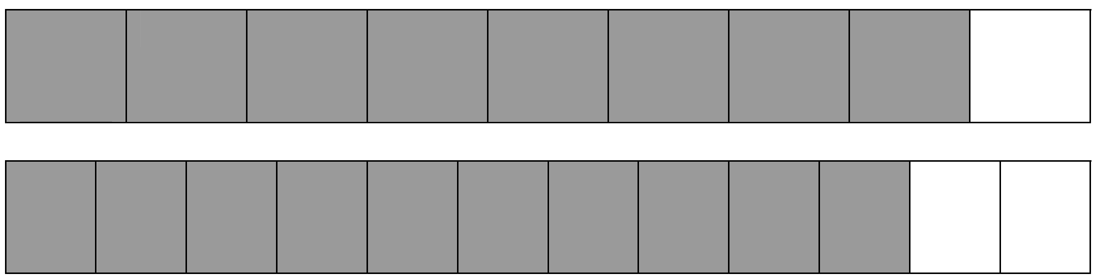
Insert $<$, $>$, or $=$.

$\frac{4}{11} \square \frac{8}{11}$ $\qquad$ $\frac{12}{22} \square \frac{10}{22}$

B. Unlike Denominators

It is a bit more tricky when the denominators are not the same.

1. AREA MODEL $\qquad \frac{8}{9} \square \frac{10}{12}$



2. MAKE SAME DENOMINATORS AND COMPARE

$\frac{5}{6}$ and $\frac{7}{9}$ $\qquad$ $\frac{-2}{5}$ and $\frac{-1}{3}$ $\qquad$ $\frac{17}{32}$ and $\frac{19}{40}$

3. USING CROSS-PRODUCTS

THEOREM	If $a, b, c,$ and $d,$ are integers and $b > 0$, $d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if, and only if, $ad > bc$

$\frac{3}{7}$ and $\frac{5}{12}$ $\qquad$ $\frac{-9}{12}$ and $\frac{-4}{5}$

EXAMPLE 1: Order the following fractions from least to greatest:

$\frac{3}{4}, \frac{9}{16}, \frac{5}{8}$, and $\frac{2}{3}$

EXAMPLE 2 $\frac{4}{7}, \frac{7}{13}$, and $\frac{14}{25}$

VII. DENSENESS OF RATIONAL NUMBERS

DENSENESS PROPERTY FOR RATIONAL NUMBERS

Given rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, there is another rational number between these two numbers.

There is **no** "next" rational number after a given rational number. Rational numbers are **not** discrete

Find ***one*** rational number between $\frac{2}{3}$ and $\frac{1}{2}$.

Find ***two*** rational numbers between $\frac{7}{18}$ and $\frac{1}{2}$.

QUESTION? How many rational numbers lie between $\frac{1}{4}$ and $\frac{3}{4}$ (or between **any** two rational numbers)?