M120 Review Functions of Several Variables 4/10

1) A petroleum company has a Cobb–Douglas production function $f(x, y) = 70x^{2/5}y^{3/5}$ where x is the utilization of labor and y is the utilization of capital. Determine the number of units of petroleum produced when 1200 units of labor and 2100 units of capital are used.

2) Let $f(x, y) = \frac{1}{2}x^2 \cdot e^{x/y}$ .Find $\frac{\partial f}{\partial y}$.

3) Let $f(x, y) = x^3y + e^{x + 3y}$. Compute f_{yx}.

4) A certain manufacturer can produce $f(x, y) = 10(6x^3 + y^2)$ units of goods by utilizing x units of labor and y units of capital. Which of the following gives the **marginal** productivity of labor when $x = 10$ and $y = 20$?

5) Let $f(x, y) = 4y^2 - 2x^3 + 5xy^2$. Find $\frac{\partial f}{\partial x}$.

6) Let $f(x, y) = x^2e^{xy}$. Find $\frac{\partial f}{\partial x}$.

7) Let $f(x, y) = \ln(x + 2y)$. Calculate f_{yx}.

8) Let $f(x, y) = x^2y + y^2x + 2xy$. Calculate f_{xx}, f_{xy}, f_{yx}, f_{yy}.

9) Let $f(x, y) = \frac{x}{y + 1}$. Calculate f_{yx}.

10) Let $f(x, y) = 5x^2 - 5y^2 + 2xy + 34x + 38y + 12$.
   a) At which point(s) does $f(x, y)$ have a *possible* extreame value?
   b) Identify each point as a relative maximum, minimum, or saddle point.

11) Let $f(x, y) = 2x^2 + 2y^3 - x - 6y + 14$
   a) At which point(s) does $f(x, y)$ have a *possible* extreame value?
   b) Identify each point as a relative maximum, minimum, or saddle point.

12) Let $f(x, y) = xy - 2x^2 + x - 4y + 1$.
   a) At which point(s) does $f(x, y)$ have a *possible* extreame value?
   b) Identify each point as a relative maximum, minimum, or saddle point.

13) A tennis racket manufacturer produces two types of rackets, standard and competition. The weekly revenue function, in dollars, for x standard rackets and and y competition rackets is given by

$$R(x, y) = 54x + 2xy + 398y - 2x^2 - 9y^2$$

   i) How many of each type of racket must be produced each week to maximize revenue?
   ii) What is the maximum weekly revenue?

14) Calculate the iterated integral $\int_{-1}^{2}\left(\int_{-2}^{1} 32x^3y^3\, dy\right) dx$.

15) Calculate the iterated integral $\int_{1}^{2}\int_{2}^{4} \frac{x}{y}\, dy\, dx$.

16) Let R be the rectangle consisting of all points (x, y) such that $1 \le x \le 16$, $1 \le y \le 9$. Calculate $\int\int_R 6\sqrt{xy}\ \ dx\, dy$.

17) Let R be the rectangle consisting of all points (x, y) such that $2 \le x \le 3$, $0 \le y \le 2$. Calculate $\int\int_R (x + y)\ \ dy$

dx. Enter just an integer.

18) The table lists the high school grade-point averages of six students and their college grade-point averages after one year of college.

High School GPA	College GPA
2.1	1.6
2.4	1.9
2.7	2.3
3.0	2.5
3.3	2.7
3.8	3.4

a) Find the least-squares line to best fit this data, where college GPA is a function of h.s GPA.
b) Use the least-squares line to estimate the college GPA for a student with a high school GPA of 3.5.

Answer Key
Testname: REVIEW MULTIVARIABLE FUNCTIONS

1) 117,517 units
2) $-\frac{x^3}{2y^2}e^{x/y}$

3) $3x^2 + 3e^{x+3y}$
4) 18,000 units
5) $-6x^2 + 5y^2$
6) $2xe^{xy}+x^2ye^{xy}$
7) $\frac{-2}{(x+2y)^2}$
8) $f_{xx} = 2y$; $f_{xy} = f_{yx} = 2x + 2y + 2$; $f_{yy} = 2x$
9) $\frac{-1}{(y+1)^2}$
10) a) (−4, 3) b) Saddle Point
11) $\left(\frac{1}{4}, 1\right)$ Relative Minimum, $\left(\frac{1}{4}, -1\right)$ Saddle Point
12) (4, 15) Saddle Point
13) i) 26 standard rackets and 25 competition rackets;
ii) $5677

14) −450
15) $\frac{3}{2}\ln 2 \approx 1.04$
16) 4368
17) 7
18) a) y = 1.0166x − 0.5312; b) about 3.0