

A SHORT TABLE OF INTEGRALS

Forms Involving  $a + bu$

1.  $\int \frac{u du}{a + bu} = \frac{1}{b^2} [a + bu - a \ln |a + bu|] + C$
2.  $\int \frac{u^2 du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$
3.  $\int \frac{u du}{(a + bu)^2} = \frac{1}{b^2} \left[ \frac{a}{a + bu} + \ln |a + bu| \right] + C$
4.  $\int \frac{u du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a) \sqrt{a + bu} + C$
5.  $\int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C \quad a > 0$
6.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
7.  $\int \frac{du}{u^2(a + bu)} = \frac{-1}{a} \left[ \frac{1}{u} + \frac{b}{a} \ln \left| \frac{u}{a + bu} \right| \right] + C$
8.  $\int \frac{du}{u^2(a + bu)^2} = \frac{-1}{a^2} \left[ \frac{a + 2bu}{u(a + bu)} + \frac{2b}{a} \ln \left| \frac{u}{a + bu} \right| \right] + C$

Forms Involving  $\sqrt{a^2 + u^2}$

9.  $\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln |u + \sqrt{a^2 + u^2}| + C$
10.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln |u + \sqrt{a^2 + u^2}| + C$
11.  $\int \frac{du}{u \sqrt{a^2 + u^2}} = \frac{-1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
12.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$
13.  $\int u^2 \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln |u + \sqrt{a^2 + u^2}| + C$

Forms Involving  $\sqrt{a^2 - u^2}$

14.  $\int \frac{du}{u \sqrt{a^2 - u^2}} = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$
15.  $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = \frac{\sqrt{a^2 - u^2}}{a^2 u} + C$
16.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{a + u}{a - u} \right| + C$
17.  $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

Forms Involving  $\sqrt{u^2 - a^2}$

18.  $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$
19.  $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = \frac{-\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$
20.  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$
21.  $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

Forms Involving  $e^{au}$  and  $\ln u$

22.  $\int u e^{au} du = \frac{1}{a^2} (au - 1) e^{au} + C$
23.  $\int \ln u du = u \ln |u| - u + C$
24.  $\int \frac{du}{u \ln u} = \ln |\ln |u|| + C$
25.  $\int u^m \ln u du = \frac{u^{m+1}}{m+1} \left( \ln u - \frac{1}{m+1} \right) \quad m \neq -1$

Reduction Formulas

26.  $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$
27.  $\int (\ln u)^n du = u (\ln u)^n - n \int (\ln u)^{n-1} du$
28.  $\int u^n \sqrt{a + bu} du = \frac{2}{b(2n+3)} [u^n(a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} du] \quad \text{for } n \neq -\frac{3}{2}$