M120 Comprehensive Final Review (rev 2/08)

Unit 1 - Limits, Continuity, Expenentials & Logs

1. Find each limit, if it exists.

   a) $\lim_{x \to -1} \frac{6x+5}{5x-6}$ b) $\lim_{x \to -3} \frac{x-3}{x^2-6x+9}$ c) $\lim_{x \to -2^-} \frac{8x}{x+2}$

2. Find the limit:

   a) $\lim_{x \to 2^-} \frac{|x-2|}{x-2}$ b) $\lim_{x \to 2^+} \frac{|x-2|}{x-2}$ c) $\lim_{x \to 2} \frac{|x-2|}{x-2}$

3. The concentration of caffeine found in Mr. Cole's blood stream t minutes after finishing his morning cup of coffee is given by: $C(t) = \frac{2t}{t^2+12}$. Find $\lim_{t \to \infty} C(t)$.

4. Differentiate: a) $y = e^x$ b) $y = \ln x$

5. Differentiate: a) $y = \frac{\ln x}{x^3}$ b) $f(x) = \ln(x^2 - 2)^{2/3}$

6. Differentiate: a) $y = e^{-2x+x^2}$ b) $y = x^2 e^{-x}$

Integration

7. Find: $\int (5e^x + 8x^3 - 2\sqrt[3]{x^4} - 6\sqrt{x})dx$

8. Find: $\int (3x - \frac{1}{x} + 7)dx$

9. Find: $\int (\frac{x^2+1}{x^2})dx$

10. Find: $\int \frac{x}{(3x^2-3)^4} dx$

11. Find: $\int \ln(4x)dx$

12. A defense contractor is starting production on a new missile control system. On the basis of data collected while assembling the first 16 control systems, the production manager obtained the following function describing the rate of labor use: $L'(x) = \frac{2000}{\sqrt[3]{x}}$ and if the first 8 control units require 12,000 labor-hours,

    a) Find the labor-hours function $L(x)$

    b) How many labor-hours will be required for the first 27 control units?

13. The management of an oil company estimates that oil will be pumped from a producing field at a rate given by $R(t) = \frac{56}{\sqrt{t+7}}$ for $0 \le t \le 20$, where $R(t)$ is the rate of production in thousands of barrels per year, t years after pumping begins. How many barrels of oil, $Q(t)$, will be produced during the first 6 years?

For the following definite integrals, show $F(x)\Big|_a^b$ and evaluate.

14. Evaluate: $\int_1^2 (5 - \frac{16}{x^3} + \frac{3}{x})dx$

15. Evaluate: $\int_0^1 (e^{0.3x})dx$

16. Evaluate: $\int_1^5 \frac{x}{x^2+1}\, dx$

17. Evaluate the improper integral: $\int_0^\infty \frac{3x}{x^2+1}\, dx$

18. Evaluate the improper integral: $\int_0^{\infty} 3e^{-5x}\, dx$

More Integration

Sketch a graph of the region and SET UP the integral. Use calculator to evaluate the integral

19. Find the area bounded by $f(x) = x^2 - x$ and $y = x + 3$.

20. Find the area of the region bounded by $f(x) = x^3 - 3x^2 + 3x$ and $g(x) = x^2$

Use "rename with algebra", substitution, or parts to find the following integrals.

21. Find $\int (\ln 8x) dx$

22. Find $\int \frac{e^x}{1+e^x} dx$

23. Find $\int \frac{6x+2\sqrt{x}}{x}\, dx$

24. Find $\int xe^{-2x}\, dx$

25. Find $\int x^3 e^{-3x}\, dx$ using the "shortcut" technique.

Functions of Two Variables

26. For $f(x, y) = \frac{x^2}{y} - \frac{y^2}{x}$, find: a) $f_x(x, y)$ b) $f_y(x, y)$

27. For $z = f(x, y) = e^{3x^3+2y^2}$, find: a) $f_x(x, y)$ b) $f_y(x, y)$

28. For $f(x, y) = x^2 + y^3 - 2xy^2$, find: a) f_{xx} b) f_{yx} c) f_{xy} d) f_{yy}

29. For $f(x, y) = e^x + 2e^y + 3xy^2 + 1$, find: a) $f_{xx}(x, y)$ b) $f_{yx}(x, y)$

30. The profit function for sales of two models of television sets at a chain discount store is given by $P(x, y) = 140x + 160y - 6x^2 + 4xy - 8y^2 - 500$ where x is the number of sales per week of model A, and y is the number of sales per week of model B.
a) Find $P_x(x, y)$ b) Find and interpret $P_x(10, 15)$

<u>The Second Partials Test</u>

If $f(a, b)$ is a critical point, then let $D(x, y) = f_{xx}(a, b) f_{yy}(a, b) - \left(f_{xy}(a, b) \right)^2$

If $D < 0$, then $f(a, b)$ is a saddle point

If $D > 0$, and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum.

If $D > 0$, and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum.

If $D = 0$, then the test is inconclusive

31. Find the critical points of the given function and classify them as relative maxima, relative minima, or saddle points: $f(x, y) = x^3 - 3x^2 - 9x + (y - 2)^2$

32. A company that manufactures tennis rackets has determined that the demand equations for the two types of rackets they produce are given by $p = 54 - 2x + y$ and $q = 398 + x - 9y$ where p is the price of the standard tennis racket, q is the price of the competition tennis racked, x is the weekly demand for standard rackets, and y is the weekly demand for competition rackets. How many of each type of racket must be produced to maximize revenue.

33. Evaluate: $\int_0^3 \int_0^1 (4xy + 12x^2y)\ dydx$

34. Evaluate $\iint\limits_R ye^x dA$ for $R = \{(x,y) | 0 \le x \le 2, -2 \le y \le 3\}$

35. Write using summation notation: $\frac{1}{3} + \frac{2}{5} + \frac{3}{7} + \frac{4}{9} + \ldots$

36. Write an expression for the nth term of the given sequence: $\frac{8}{3}$, $\frac{10}{3}$, 4, $\frac{14}{3}$, ...

37. Does this geometric series converge? If yes, find its sum:

a) $\sum_{n=1}^{\infty} e^{-0.2n}$ b) $\sum_{n=2}^{\infty} \left(-\frac{4}{3}\right)^n$ c) $\sum_{n=1}^{\infty} \left(\frac{3^n}{4^n}\right)$

38. Determine whether the given series converges or diverges and indicate why.

a) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right)$

c) $\sum_{n=1}^{\infty} \left(\frac{10^n}{n!}\right)$ d) $\sum_{n=1}^{\infty} \left(ne^{-n^2}\right)$

39. Find a Taylor polynomial of degree 3 to estimate e^{-3x} near zero .

Solutions

1. a) $\frac{1}{11}$ b) $-\frac{1}{6}$ c) ∞

2. a) -1 b) 1 c) Does not exist

3. Approaches 0

4. $a)\ e^x$ $b)\ \frac{1}{x}$

5. $a)\ \frac{1-3\ln x}{x^4}$ $b)\ \frac{4x}{3(x^2-2)}$

6. $a)\ (-2+2x)e^{-2x+x^2}$ or $2(-1+x)e^{-2x+x^2}$ $b)\ 2xe^{-x} - x^2e^{-x}$ or $xe^{-x}(2-x)$

7. $5e^x + 2x^4 - \frac{6}{7}\sqrt[3]{x^7} - 4\sqrt{x^3} + C$

8. $\frac{3}{2}x^2 - \ln|x| + 7x + C$

9. $x - \frac{1}{x} + C$

10. $-\frac{1}{18(3x^2-3)^3} + C$

11. $x\ln(4x) - x + C$

12. a) $L(x) = 3000\sqrt[3]{x^2}$ b) $Ans : L(27) = 27,000$

13. $\int_0^6 56(t+7)^{-\frac{1}{2}}dx \sim 107.5$

14. $5x + \frac{8}{x^2} + 3\ln x\Big]_1^2 \sim 1.08$

15. $\frac{10}{3}e^{0.3x}\Big]_0^1 \sim 1.166$

16. $\frac{1}{2}\ln|x^2+1|\Big]_1^5 \sim 1.28$

17. Diverges

18. Converges to $\frac{3}{5}$

19. $\int_{-1}^{3}\Big((x+3)-(x^2-x)\Big)dx = 10\,\frac{2}{3}$

20. $\int_0^1\Big((x^3-3x^2+3x)-x^2\Big)dx+\int_1^3\Big(x^2-(x^3-3x^2+3x)\Big)dx \sim .416+2.66$ or $3\,\frac{1}{12}$

21. Parts : $u=\ln x;\ dv=1$ $\qquad x\ln 8x - x + C$

22. Subst : $u=1+e^x$ $\qquad \ln|1+e^x|+C$

23. Algebra/rename $\int(6+2x^{-\frac{1}{2}})dx$ $\qquad 6x + 4\sqrt{x} + C$

24. Parts : $u=x\quad dv=e^{-2x}$ $\qquad -\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$

25. $-\frac{1}{3}x^3e^{-3x}-\frac{1}{3}x^2e^{-3x}-\frac{2}{9}xe^{-3x}-\frac{2}{27}e^{-3x}+C$

26. $a)\ \frac{2x}{y}-\frac{y^2}{x^2}$ $\qquad b)\ -\frac{x^2}{y^2}-\frac{2y}{x}$

27. $a)\ 9x^2e^{3x^3+2y^2}$ $\qquad b)\ 4ye^{3x^3+2y^2}$

28. $a)\ 2$ $\qquad b)\ -4y$ $\qquad c)\ -4y$ $\qquad d)\ 6y-4x$

29. $a)\ e^x$ $\qquad b)\ 6y$

30. $a)$ $140-12x+4y$ b) 80

31. Min of -27 at $(3,2)$ Saddle Point of 5 at $(-1,2)$

32. $R(x,y)=xp+yq=54x-2x^2+2xy+398y-9y^2$
26 standard rackets, 25 competition rackets for Revenue of 5677

33. $(x^2+2x^3)\Big|_0^3\ or\ (126y)\Big|_0^1 = 63$

34. ~ 15.97

35. $\sum_{n=1}^{\infty}\frac{n}{2n+1}$

36. $a_n=\frac{2n+6}{3}$ or $\frac{2(n+3)}{3}$

37. a) $\frac{e^{-0.2}}{1-e^{-0.2}} \sim 4.517$ b) No. $|r|>1$ c) $\frac{3/4}{1-3/4}=3$

38. a) Diverge; nth term test =1 not 0 $\qquad$ b) Converge, p-series; p=2>1

c) Converge, ratio Test $\qquad$ d) Converge - Integral Test

39. $p_3=1-3x+\frac{9}{2}x^2-\frac{9}{2}x^3$