M119 Chapter 2 Review (rev 9/09)

1. State the definition of the derivative and use the definition to find the derivative of $f(x) = 2x^2 - 5x + 3$

2. Find the derivative at the point when x =1, and explain the meaning of this number. $f(x) = 4x^2 - x$

3. Find the slope of the line tangent to the graph of $f(x) = \frac{x-2}{x^2-1}$ at $x = -2$

4. Find the equation of the line which is tangent to the graph of $f(x) = \sqrt{2x+9}$ at $x = 0$

5. What is the rate of change of f(t) with respect to t when t = 1 if $f(t) = t^2 + \frac{3}{t} - \frac{1}{\sqrt{t}}$

6. Find $\frac{dy}{dx}$. Express answer as a function of *x* only. $y = \frac{1}{2}u^2 + 2u^3;\ u = 1 - 3x$

7. Find $\frac{dy}{dx}$ for the given value of x. $y = \frac{4}{3}u^3 - 5u^2 + 3u;\ u = 2x^2 - 3x;\ x = 1$

8. Differentiate: $f(x) = \sqrt{x^3 - 2x + 1}$

9. Find the point(s) on the curve where the derivative is zero: $f(x) = \left(\frac{x}{x+1}\right)^2$

10. Differentiate and simplify: $y = \frac{(x+2)^4}{(2-x)^3}$

11. Find the third derivative of $f(x) = 3x^2 + 4x + 5 - \frac{1}{\sqrt[3]{x}} + \sqrt{x}$

12. A manufacturer estimates that the cost of manufacturing *x* units of a particular product will be $C(x) = \frac{1}{2}x^2 + 4x + 66$ dollars.
    a) Find the marginal cost.
    b) Use marginal cost to estimate the cost of the 26th unit.
    c) What is the actual cost of producing the 26th unit?

13. It is projected that t months from now, the population of a certain town will be $P(t)=3t+5\sqrt{t^3}+6000$. At what percentage rate will the population be changing with respect to time 4 months from now?

14. Suppose the weight in grams of a cancerous tumor at time t is $W(t)=0.1t^2$, where t is measured in weeks.
    a) What is the rate of growth of the tumor after 5 weeks?
    b) At what time is the tumor growing at the rate of 5 grams per week?

15. A manufacturer's total monthly revenue is $R(q)=300q+0.03q^2$ dollars when q units are produced during the month. Currently the manufacturer is producing 70 units a month and is planning to decrease the monthly output by 0.75 unit. Estimate how the total monthly revenue will change as a result.

16. The Gross Revenue of a company was $G(x)=2x^2+12x+3$ million dollars x years after 1992. Use calculus to estimate the percentage change in the Gross Revenue during the 1st quarter of 1995.

17. When a certain commodity is sold for p dollars per unit, consumers will buy $D(p)=\frac{40,000}{p}$ units per month. It is estimate that t months from now, the price of the commodity will be $p(t)=0.4\sqrt{t^3}+6.8$ dollars per unit. At what rate will the monthly demand for the commodity be changing with respect to time 4 months from now?

18. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 a.m. will have produced $Q(t)=-t^3+9t^2+12t$ units t hours later.
    a) Compute the worker's rate of production at 10:00 a.m.
    b) At what rate is the worker's rate of production changing with respect to time at 10:00 a.m.?

Solutions

1. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$; $f'(x)=4x-5$
2. $f'(x)=8x-1$; $f'(1)=7$ is the slope of the line tangent to the graph at (1,3)
3. $f'(x)=\frac{-x^2+4x-1}{(x^2-1)^2}=\frac{-13}{9}$
4. $f'(x)=\frac{1}{2}(2x+9)^{-1/2}\bullet 2=\frac{1}{\sqrt{2x+9}}$; $f'(0)=\frac{1}{3}$; $f(0)=3$; $y=\frac{1}{3}x+3$
5. $f'(t)=2t-3t^{-2}+\frac{1}{2}t^{-3/2}=2t-\frac{3}{t^2}+\frac{1}{2\sqrt{t^3}}$; $f'(1)=-\frac{1}{2}$
6. $-3(1-3x)-18(1-3x)^2$ or $-3(1-3x)(7-18x)$
7. $dy/dx=4(2x^2-3x)^2(4x-3)-10(2x^2-3x)(4x-3)+3(4x-3)$; $f'(1)=17$
8. $f'(x)=\frac{1}{2}(x^3-2x+1)^{-1/2}(3x^2-2)=\frac{3x^2-2}{2\sqrt{x^3-2x+1}}$
9. $f'(x)=2\left(\frac{x}{x+1}\right)\left(\frac{1(x+1)-1(x)}{(x+1)^2}\right)=\frac{2x}{(x+1)^3}$; $\frac{2x}{(x+1)^3}=0\Rightarrow x=0$; $(0,0)$
10. $\frac{dy}{dx}=\frac{4(x+2)^3\cdot 1\cdot(2-x)^3-3(2-x)^2(-1)\cdot(x+2)^4}{\left((2-x)^3\right)^2}=\frac{(x+2)^3(14-x)}{(2-x)^4}$
11. $f'''(x)=\frac{28}{27\sqrt[3]{x^{10}}}+\frac{3}{8\sqrt{x^5}}$
12. a) $C'(x)=x+4$ b) \$29 c) C(26) – C(25) = \$29.50
13. $\frac{P'(4)}{P(4)}*100\%=\frac{18}{6052}*100\%=0.297\%/\text{month}$
14. a) 1 gram/week b) 25 weeks
15. $\Delta R=R'(70)\cdot(-.75)=(300+.06\cdot 70)\cdot(-.75)=228.15$ decrease by \$228.15
16 $\frac{.25\times G'(3)}{G(3)}\times 100\%=\frac{6}{57}\times 100\%=10.53\%$
17. $D'(t)=-40000(.4t^{3/2}+6.8)^{-2}(0.6\sqrt{t})$; so decrease at rate of 480 units/month
18. a) $Q'(2)=$ 36 units/hour b) $Q''(2)$ 6 units/hour/hour