5.5 CENTRAL LIMIT THEOREM

An Experiment:

Each student generates a set of 40 random numbers from 0-9, entering the numbers in a list. The mean and standard deviation of the set of 40 numbers are calculated. The results are then recorded on a master sheet. The class totals for numbers 0...9 will be plotted and μ and σ calculated. The Averages will also be plotted and $\mu_{\overline{x}}$ and $\sigma_{\overline{x}}$ calculated. The histogram of the class totals will be compared to the histogram of the sample means.

"As the sample size increases, the sampling distributions of sample means approaches a normal distribution."

See example on page 261-2 for results of a similar experiment.

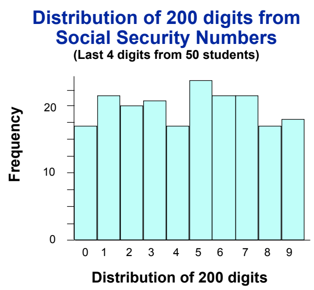


Figure 5-19

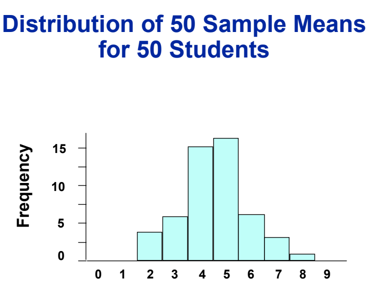


Figure 5-20

SAMPLING DISTRIBUTION OF THE MEAN: The probability distribution of sample means, with all samples having the same size n.

CENTRAL LIMIT THEOREM

Given: 1. The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation σ.

2. Samples of size n are randomly selected from this population of x values. (The samples are selected so that all possible samples of size n have the same chance of being selected.)

Conclusions:

1. The distribution of sample means $\bar{x}$ will, as the sample size increases, approach a normal distribution.

2. The mean of the sample means ($\mu_{\overline{x}}$) will be the population mean μ. (That is, the normal distribution from Conclusion 1 has mean μ.)

3. The standard deviation of the sample means ($\sigma_{\overline{x}}$) will be $\frac{\sigma}{\sqrt{n}}$. (That is, the normal distribution from Conclusion 1 has standard deviation $\frac{\sigma}{\sqrt{n}}$.)

Practical Rules:

1. For samples of size n larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n gets larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for any size n (not just those larger than 30).

NOTATION

Mean of the Sample Means $\boldsymbol{\mu_{\overline{x}} = \mu}$

Standard Deviation of the Sample Means $\boldsymbol{\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}}$

often called STANDARD ERROR OF MEAN

RULES WHEN USING THE CENTRAL LIMIT THEOREM:

- If you are working with a random sample of size $n > 30$, or if the original population is normally distributed, treat the distribution of sample means as a normal distribution.

- The distribution of the sample means has mean μ.

- The distribution of the sample means has standard deviation $\frac{\sigma}{\sqrt{n}}$.

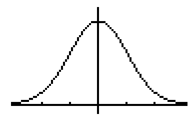
INTERPRETING RESULTS

Recall the **RARE EVENT RULE:**

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

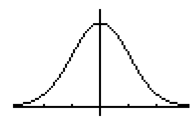
See the Body Temperatures example on page 265 which illustrates the type of thinking that is the basis for hypothesis testing in Chapter 7.

Examples:



***#4, pg. 267 a)A single individual
(a 5.3 problem)

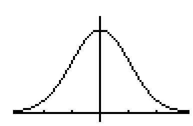
$P(100 < x < 165)$



b) Selecting a SAMPLE of 81 (Central Limit Theorem)

$P(100 < \overline{x} < 165)$

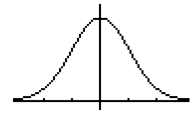
***#6, pg. 267



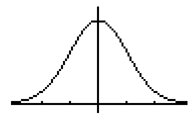
a)

b)

***#16, pg.269 a) P($x > 590$)



b) $P(\overline{x} > 590)$



c)

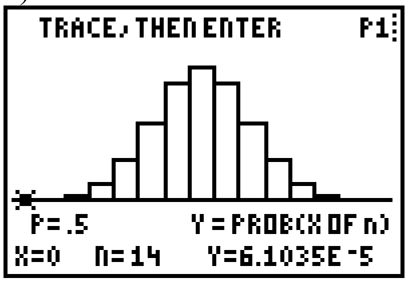
d)

<u>Due the day we do 5.6 in class</u> Name___________________

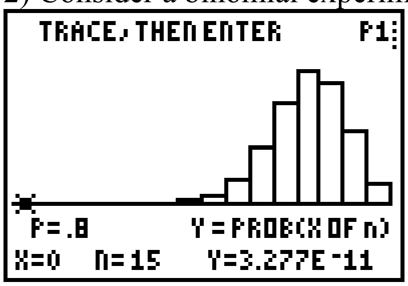
Reviewing section 4.3 about BINOMIAL PROBABILITY DISTRIBUTIONS which apply to a DISCRETE random variable.

The distributions are shown in each case. Use the methods from 4.3 to answer .

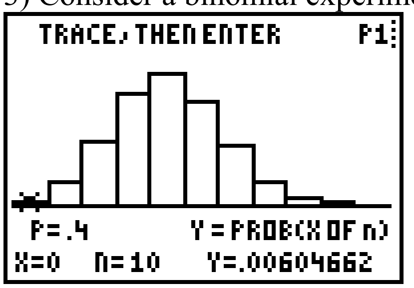
1) If we toss a coin 14 times what is the probability of tossing 8 heads?



2) Consider a binomial experiment with n = 15, and p = .8, and find $P(x \geq 8)$



3) Consider a binomial experiment with n = 10 and p = .4, and find $P(x = 7)$



4) Consider a binomial experiment with n = 14 and p = .6, and find $P(x < 9)$