T101 SECTION 4.3 FUNCTIONS

What is a *function*? It is a *relationship between numbers.* Beginning with one number and *transforming* it into another.

A **function** from a set A, called the _____ (the *inputted* numbers) to a set B, called the _____ (the *transformed or outputted* numbers) is a correspondence from A to B in which each element of A is paired with one and only one element of B. Think of this as a "rule" which assigns A to B.

Functions can be demonstrated in many different ways such as

1. Rules
2. Machines
3. Equations
4. Arrow Diagrams
5. Ordered Pairs
6. Tables
7. Graphs
8. Sequences

I. FUNCTIONS AS RULES – "GUESS MY RULE"

Student	Teacher
0	0
1	1
2	2
3	3
x	x

Student	Teacher
0	0
1	1
2	4
3	9
x	

Student	Teacher
0	3
1	5
2	7
3	9
x	

Rule _____ Rule _____ Rule _____

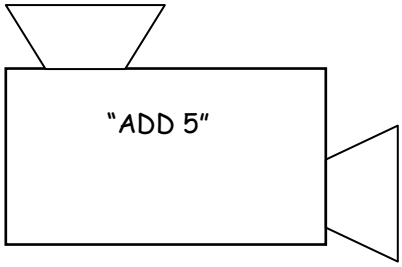
Is each of these functions (review the definition)?

Notice that all the student "inputs" are set A or the _____ and all the teacher responses or "outputs" are set B or the _____.

II. FUNCTIONS AS MACHINES

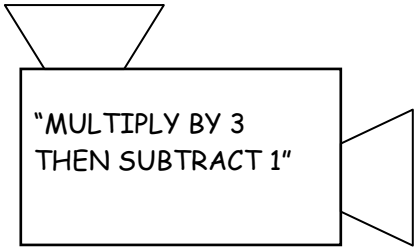
You put a number in the top "input or domain", you apply the rule, and the number that comes out is the "output or range".

DOMAIN: {1, 5, -2}



RANGE
{ }

DOMAIN: {8, 0, -7}



RANGE
{ }

III. FUNCTIONS AS EQUATIONS

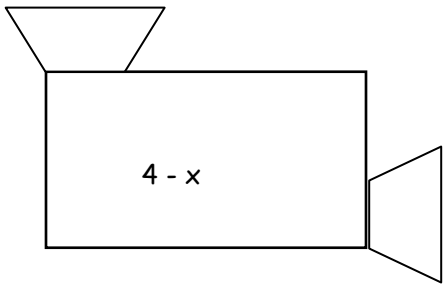
In each of the above function machine, the "rules" are functions because each element of the domain is transformed into one and only one element in the range. When a rule is a function, we use ***function notation***, which is just a different way to write the rule to indicate that it is a **function**.

MACHINE 1: f(x) = _____

MACHINE 2: g(x) = _____

EXAMPLE: **f(x) = 4 - x**

DOMAIN (x's)



RANGE (f(x)'s)

Find:

f(1) = _____

f(6) = _____

f(0) = _____

f(_____) = -3

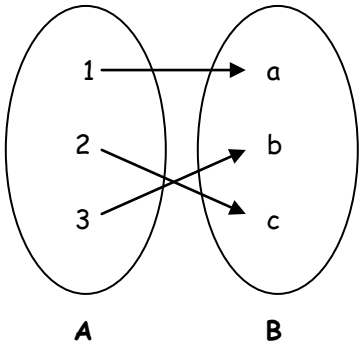
If f(x) = 3x + 7, then f(2) = _____

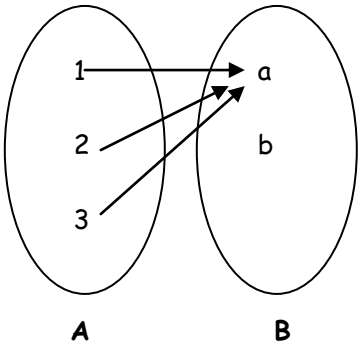
If f(x) = x^3 - 4, then f(3) = _____

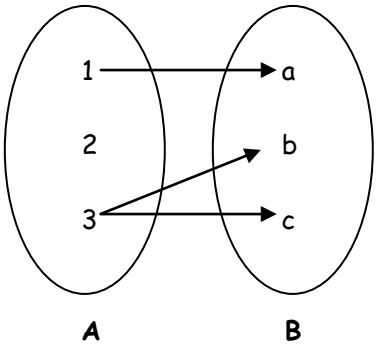
If f(x) = -5x - 18, then what value of x causes f(x) = -278?

IV. FUNCTIONS AS ARROW DIAGRAMS

Which of the following exhibit a function from A to B? (Recall definition of function)







V. FUNCTIONS AS TABLES & ORDERED PAIRS

Which of the following sets of ordered pairs represents a function?

{ (2, 5), (3, 6), (5, 8), (10, 13) }

{(1, 1), (1, -1), (4, 2), (4, -2) }

{ (2, 7), (3, 7), (4, 7), (5, 7) }

{ (2, 3), (3, 7), (4, -7), (2, 3) }

Example: A plumber charges $60 for a house call and then $45 per hour for labor. Make a table showing costs from 0 - 5 hours.

	Cost
0	
1	
2	
3	
4	
5	
h	

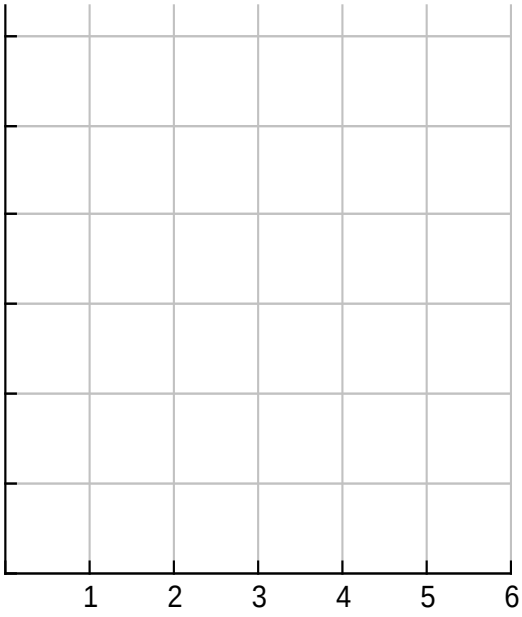
Now, write a cost function C(x) that gives the total cost in terms of x hours of labor:

C(x) = _____

VI. FUNCTIONS AS GRAPHS

The above table represents the ordered pairs in the form (hours worked, cost). List these pairs:

Now graph these ordered pairs and draw an appropriate graph. THINK ABOUT IT!



VII. SEQUENCES AS FUNCTIONS

Arithmetic, geometric and other sequences can be thought of as functions as well. The inputs (domain) would be the natural numbers and the outputs (range) would be the terms of the sequence.

Example: $a_n = 3n^2 - 2$ *could now be written as*
$f(n) = 3n^2 - 2$ (where n is the number of the particular term and f(n) is the value of the nth term)
Find the first 5 terms of this sequence.

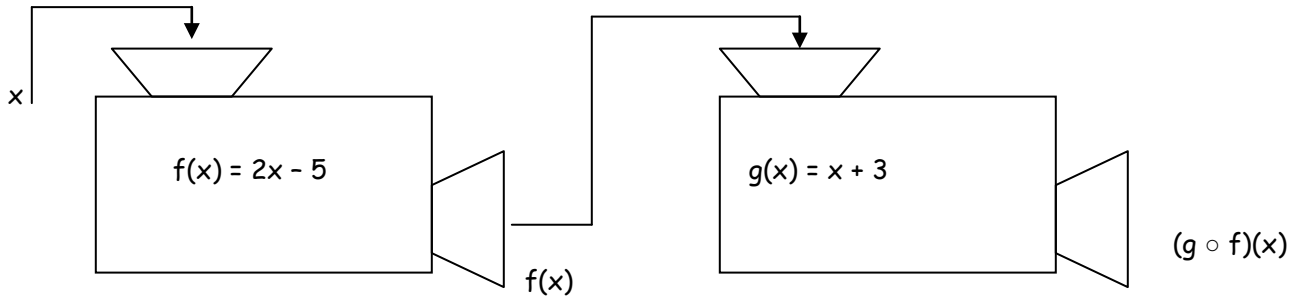
Example: Find a function f(n) that describes each of the following:

a. An arithmetic sequence whose first term is 4 and whose difference if 3

b. A geometric sequence whose first term is 3 and whose ratio is 2

VIII. COMPOSITIONS OF FUNCTIONS

The composition of two functions is "new function" formed by operating on each in a specific order. The domain is inputted into the first function, the range is then outputted. This range then becomes the domain of the second function, which finally outputs the final range. Examine the following two function machines:



The first function f is followed by a second function g. We symbolize this composition as:

_____ or _____

If we input 3 as the domain, we symbolize the output or range as:

_____ or _____ which would equal _____.

If f(x) = -4x - 1 and g(x) = x + 3, compute the following:

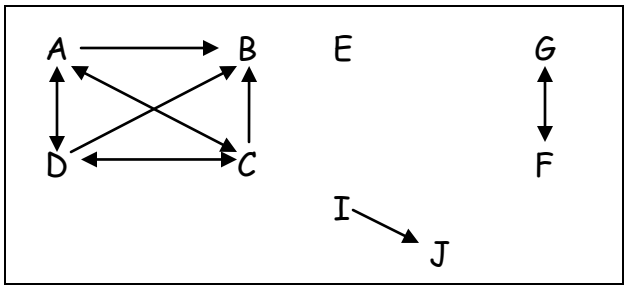
(g ∘ f)(2) = (f ∘ g)(2) =

(g ∘ f)(x) = (f ∘ g)(x) =

IX. RELATIONS

A relation from set A to set B is a correspondence between elements of A and elements of B, but unlike functions, we do not require that each element of A be paired with one, and only one, element of B. *Thus any ordered pair is a relation.*

The relation in the following diagram is *"is the sister of"*.
So if Sue is the sister of Tom, this would be represented by: S ⟶ T
But, if Sue is the sister of Tina, this would be represented by: S ⟷ T Why?



Using the following diagram, list the letters that indicate,
Who is a girl? _____ Who is a boy? _____ Which are undeterminable? _____

We could also show this relation *"is the sister of"* using ordered pairs.
{ (A, B), (A, C), (A, D)_____ }

PROPERTIES OF RELATIONS

REFLEXIVE PROPERTY
A relation R on a set X is ***reflexive*** if, and only if, for every element $a \in X$, a is related to a That is, for every $a \in X$, $(a, a) \in R$.

SYMMETRIC PROPERTY
A relation R on a set X is ***symmetric*** if, and only if, for every element a and b *in* X, whenever a is related to b, then b is also related to a. That is, if $(a, b) \in R$, *then* $(b, a) \in R$.

TRANSITIVE PROPERTY
A relation R on a set X is ***transitive*** if, and only if, for all elements a, b *and* c *in* X, whenever a is related to b, and b is related to c, then a is also related to c. That is, if $(a, b) \in R$, and $(b, c) \in R$, *then* $(a, c) \in R$.

EQUIVALENCE RELATION
Any relation, R, that satisfies the reflexive, symmetric, and transitive properties.

Tell which of the following is Reflexive, symmetric, or transitive on the set of all people. Which are equivalence relations?

a. "Is younger than" b. "Is the same weight as" c. "Is the neighbor of"

T101 SECTION 4.3 FUNCTION WORKSHEET

1. Given the following function: $f(x) = 2x - 5$
Find:

a. $f(1)$

b. $f(-2)$

c. $f(k)$

d. $f(-k)$

g. $f(k + 2)$

2. Given the following function: $f(x) = x^2$
Find:

a. $f(8)$

b. $f(-5)$

c. $f(w)$

d. $f(-w)$

e. $f(w + 4)$

3. If $f(x) = x - 3$ **and** $g(x) = 2x$
Find:

a. $(f \circ g)(4)$

b. $(g \circ f)(-5)$

c. $(f \circ g)(k)$

d. $(g \circ f)(k)$

e. $(f \circ g)(k + 1)$

f. $(g \circ f)(4k)$

4. Let $f(n) = 2n - 1$ represent the n^{th} term of a sequence - where n is a natural number only. Which of the following are values of the function? If so, for what value of n?

a. 45

b. –17

c. 22

d. 999

5. Let $f(n) = n^2$ represent the n^{th} term of a sequence for - where n is a natural number only. Which of the following are values of the function? If so, for what value of n?

a. 1

b. –25

c. 400

d. 33