T101 SECTION 5.5 GREATEST COMMON DIVISOR *AND* LEAST COMMON MULTIPLE

What is *the greatest number that divides both 24 and 30.*

This is also known as:

____ or ___________.

DEFINITION:	The **greatest common divisor (GCD)** of two integers *a* and *b* is the greatest integer that divides both *a* and *b*. **Notation: GCD (*a*, *b*)**

There are many methods that we can use to find the GCD. Here are a few:

<u>COLORED RODS MODEL (CUISENAIRE RODS)</u>

To find the GCD, find the longest rod such that you can use multiples of that rod to build ***both*** of the numbers.

Example: Find the GCD of 6 and 8 – written as GCD (6, 8)

<u>INTERSECTION-OF-SETS METHOD</u>

List all the divisors of each number, find the intersection of these sets, then choose the ***greatest divisor*** *that is in that intersection.*

GCD (24, 60) Divisors of 24: ____

Divisors of 60: ____

Divisors of both 24 and 60:_____________________________________

GCD (24, 60) is ___________.

PRIME FACTORIZATION METHOD

Find the prime factorization of each number. Then circle each of the prime numbers that are ***common*** – that is on each list – be aware you may be circle more than one "2" or "3" etc. Multiply these prime factors together.

GCD (36, 180)

GCD (108, 72)

(WRITE WITHOUT EXPONENTS)

36 = ____________________________

180 = ___________________________

GCD (36, 180) = ______________

108 = ____________________________

72 = ____________________________

GCD (108, 72) = _______________

GCD ($16x^2y$, $20xy^2$)

$16x^2y$ = ________________________________

$20xy^2$ = ________________________________

GCD ($16x^2y$, $20xy^2$) = ____________________

GCD (7, 11)

7 = ________________

11 = _______________

GCD(7, 11) = _______________

Note: The GCD (0, *b*) = _____________ Why? ____.

Note: If the GCD (*a*, *b*) = 1, then *a* and *b* are referred to as being _______________________.

Give an example of any two numbers that are *relatively prime*. __________ and____________

Give an example of two composite numbers that are *relatively prime*. ________ and________

EUCLIDEAN ALGORITHM METHOD

If the numbers are very large – thereby being very difficult to factor – then we can use the following theorem to help us find the GCD.

THEOREM	If a and b are any whole numbers greater than 0 and $a \geq b$, then GCD (a, b) = GCD (b, r) where r is the remainder when a is divided by b.

GCD (546, 390)

GCD (3150, 4563)

Let's try using the Euclidean algorithm, but let's do the work in our heads:

GCD(40, 18) = GCD(______,_______)

= GCD(______,_______)

= GCD(______,_______)

= ________

GCD(36, 15) = GCD(______,_______)

= GCD(______,_______)

= GCD(______,_______)

= ________

LEAST COMMON MULTIPLE

What is *the smallest number that is a multiple of both 15 and 12.*

This is also known as:

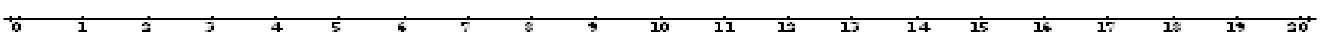
____ or __________.

DEFINITION	The **least common multiple (LCM)** of two positive integers *a* and *b* is the least positive multiple that both numbers have in common. **Notation: LCM (*a*, *b*)**

NUMBER LINE METHOD:
Show the multiples of 2 and 3 on the number line.

List the first 3 common multiples.

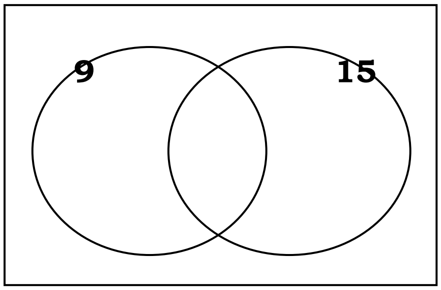
What is the LCM(2,3)?



COLORED RODS MODEL (CUISENAIRE RODS)
To find the LCM, keep building trains of each color until they are the same length. That length is the LCM.
Find the **LCM (3, 4)**

INTERSECTION-OF-SETS METHOD
List all the multiples of each number until you get one that appears on each list. That number is the LCM.
Find the **LCM (9, 15)**

Let's see the connection between GCD and LCM using a Venn Diagram:



Prime Factorization of 9 :______________

Prime Factorization of 15 :______________

Intersection (GCD):_____________________

Union (LCM):___________________________

PRIME FACTORIZATION METHOD

Find the prime factorization of each number using exponents. Choose every "type" of prime factor involved. Then write each "type" to its highest power in either factorization. Multiply these prime factors together.

LCM (36, 180)

(SEE ABOVE IN GCD)

(THIS TIME WRITE USING EXPONENTS)

36 = ___________________________

180 = __________________________

LCM (36, 180) = _____________

LCM (550, 3500)

550 = ___________________________

3500 = __________________________

LCM (550, 3500) = _____________

Find the GCD (6, 9) and the LCM (6, 9).

What is the product: GCD (6, 9) · LCM (6, 9) = ________·_________ = _________

Notice that this is the product of the two number (6 · 9 = 54)

THEOREM: For any two natural numbers *a* and *b*,

$$\mathbf{GCD}\,(a, b) \cdot \mathbf{LCM}\,(a, b) = a \cdot b$$

We can use this formula to help us find the LCM of two numbers if we knew their GCD. For instance, we found above that the GCD(3150, 4563) to be ________________. Now find the LCM (3150, 4563).

DIVISION-BY-PRIMES METHOD for finding LCM

This process involves dividing each number by the smallest possible prime that will divide **AT LEAST ONE OF THE NUMBERS (not necessarily all of them).** This method works great when you have more than two numbers.

Find LCM (12, 75, 120)

Find LCM (27, 36, 45, 60)

Janice and Bob both work night shifts. When they have the same night off, they go dancing. If Janice has every 4^{th} night off and Bob has every 7^{th} night off, how often do they go dancing?