T101 SECTION 5-4 PRIME AND COMPOSITE NUMBERS

ACTIVITY: Using 24, 1" tile squares, how many different sized rectangles can you form? (You must use all the tiles in each rectangle.) Write the different dimensions below:

The dimensions of each rectangle are the *divisors* or *factors* of 24.
Thus, 24 has how many *divisors*? ___________

What if you had 11 tile squares – how many *divisors* for 11? ________, namely _____ and _____.

Let's construct a table that shows the number of divisors for any natural number less than or equal to 20.

1. Start with 1. How many factors does 1 have?_________ Place the number 1 in that column.
2. Now do 2. How many factors does 2 have?_________ Place the number 2 in that column.

Number of Divisors (or Factors) for Natural Numbers less than 20					
Which have only 1 factor?	Which have only 2 factors?	3 factors	4 factors	5 factors	6 factors

What do notice about the numbers in the "odd-numbered" columns?

What is true about all the numbers in the "2" column?

- **DEFINITION:** Any positive integer with *exactly two distinct*, positive divisors is a ____________ number.
- **DEFINITION:** Any integer greater than 1 that has a positive factor other than 1 and itself is a ______________ number.
- Based upon the above definitions, is 1 prime or composite or neither? ____________________
- What is the smallest prime number? ______
- What is the only even prime number? _____
- Are the following are prime or composite? Why OR Why not?

1,378 3,627 65,375

PRIME FACTORIZATION

Composite numbers can be expressed as products of two or more whole numbers greater than 1.

Example: $12 = 2 \cdot 6$ or $12 =$ _____ $\cdot$ _____ or $12 =$ _____ $\cdot$ _____ $\cdot$ ______

Each of the above is called a _______________________, but the last one is special. It only contains prime numbers and is called a ____________________________________.

> **THE FUNDAMENTAL THEOREM OF ARITHMETIC**
>
> Each composite number can be written as a product of primes in ***one and only one way,*** except for the order of the prime factors in the product.

Finding the *prime factorization* using a **FACTOR TREE.**

You try:

260 or **260** **450**

Finding the *prime factorization* using a **UPSIDE DOWN DIVISION METHOD.**
(*Choose only a prime divisor for each division.*)

You try:

260 **225**

The primes in the prime factorization of a number are typically listed in increasing order from left to right and if a prime appears in a product more than once, exponential notation is used. Thus the prime factorization of:

260 is written as: _____________________

450 is written as: _____________________

225 is written as: _____________________

NUMBER OF DIVISORS

Given 81. What are its divisors? ______________________________

How many are there? _________

Its prime factorization is: ___________________________

Also written: _______________

Given 48. What are its divisors? ______________________________

How many are there? _________

Its prime factorization is: ___________________________

Also written: _______________

> **FORMULA FOR FINDING THE NUMBER OF FACTORS OF A NUMBER**
>
> In general, given a number a with prime factorization $p^n q^m$ (that is, $a = p^n q^m$),
>
> *then* a *will have* $(n + 1)(m + 1)$ *factors or divisors.*
>
> Basically what this says is that the TOTAL NUMBER of divisors (prime, composite or otherwise) for a particular number IS EQUAL TO the product of (each exponent + 1) in the prime factorization.

So, use this formula to check the number of factors for 81 and 48 from above.

$81 = 3^4$, so ______________________ = ______ factors

$48 = 2^4 \cdot 3$, so ______________________ = ______ factors

Find the number of divisors for (making good use of prime factorization and rules of exponents!):

1440 | 1,000,000 | 210^9

<u>DETERMINING IF A NUMBER IS PRIME</u>

We need to decide if 97 is a prime number.
This means that we need to find out if 97 can be divided by any number <u>between</u> 1 and 97.
Do we want to check EVERY NUMBER from 2 to 96? (WE HOPE NOT!)

Let's think? Is it necessary to divide 97 by 2, 3, 4, 5, 6, 7,…, 96 to check if it is prime or composite?

If 2 is not a divisor of 97, could any multiple of 2 be a divisor? _______________

If 3 is not a divisor of 97, could any multiple of 3 be a divisor? _______________

If 5 is not a divisor of 97, could any multiple of 5 be a divisor? _______________

If 7 is not a divisor of 97, could any multiple of 7 be a divisor? _______________

THUS, to determine if a number is prime, you must check to see if it is divisible <u>only by the prime numbers</u> less than the given number. So, what are all the prime numbers less than 97? Wow, this still is a lot to check. HOPEFULLY we can trim this list a bit more.

If we want to determine if a number is prime, we need only check the prime numbers (we said this above), but better yet, we need to only check the **<u>prime numbers whose square does not exceed the number itself</u>**.

For instance, is 109 prime or composite? (List the primes whose square is less than 109, and test those for divisibility.)

Is 91 prime or composite?

LIST PRIMES TO CHECK:

CHECK:

Is 397 prime or composite?

LIST PRIMES TO CHECK:

CHECK:

SIEVE OF ERATOSTHENES
(USED TO FIND ALL THE PRIME NUMBERS FROM 1 TO ?)

Methodically cross out all the composite numbers, thus leaving only primes.

PROBLEM SOLVING

A large toy store carries one kind of stuffed bear. On Monday the store sold a certain number of the stuffed bears for a total of $1,843 and on Tuesday, without changing the price, the store sold a certain number of the stuffed bears for a total of $1,957. How many toy bears were sold each day if the price of each bear is a whole number and greater than $1?

What do we know?

DAY	Number of Bears Sold	x	Price	=	Total Revenue
Monday		x	p	=	
Tuesday		x	p	=	

So, BOTH Monday's revenue and Tuesday's revenue are divisible by _________.

Therefore, ___________ | (1957 – 1843)

___________ | (114)

What divides 114? Make a list of possible PRICES, p.

Since p must divide BOTH $1957.00 and $1843.00, start crossing out p's above that don't pass the divisibility tests. (Remember, if $p \nmid n$, then neither will any multiple of p.)