T101 SECTION 5-3 DIVISIBILITY

Can 12 be divided by 3, evenly?

What does the word ***evenly*** mean in the previous question?

Other ways to express this:

12 is divisible by 3	a is divisible by b
3 is a divisor of 12	b is a divisor of a
12 is a multiple of 3	a is a multiple of b
3 is a factor of 12	b is a factor of a
3 divides 12	b divides a

We are going to introduce a new symbol which is shorthand for "b divides a" : $\boldsymbol{b \mid a}$

We can also alter this slightly: "a divides b" : _____

"a does not divide b" : _____

DEFINTION of "DIVIDES" If a and b are any integers, then b divides a , written $b \mid a$, if and only if, there is a unique integer c such that:

Does 6 | 12 _____, because _____

4 | 32 _____, because _____

28 | 4 _____, because _____

-5 | 30 _____, because _____

ANSWER TRUE OR FALSE:

12 | 36	5 | 127	9 | 3
7 | 42	0 | 7	7 | 0
3 | 6	3 | $6n$ (for any integer n)	$(a - b) \mid (a^2 - b^2)$

THEOREM For any integers a and d, if $d \mid a$ and n is any integer then:

(If $d \mid a$, then $d \mid$ any multiple of a)

This means that if $4 \mid 8$, then $4 \mid (8 \cdot 2)$
and $4 \mid (8 \cdot 3)$
and $4 \mid (8 \cdot 4)$ etc.

But, is the reverse true? If $4 \mid 8$, then does $(4 \cdot 2) \mid 8$
and $(4 \cdot 3) \mid 8$
and $(4 \cdot 4) \mid 8$?

THEOREM For any integers a, b, and d, the following holds

		Example
a.	If $d \mid a$ and $d \mid b$, then _____	_____
b.	If $d \mid a$ and $d \nmid b$, then _____	_____
c.	If $d \mid a$ and $d \mid b$, then _____	_____
d.	If $d \mid a$ and $d \nmid b$, then _____	_____

In other words, if $15 \mid 4500$ and $15 \mid 15$ *then*

a. $15 \mid$ _____

b. $15 \mid$ _____

ANSWER TRUE OR FALSE: ***(Look for a counterexample!)***

a. If $4 \mid a$ **and** $4 \mid b$, then $4 \mid ab$

b. If $5 \mid (x + y)$, then $5 \mid x$ **and** $5 \mid y$

c. If $12 \nmid a$, then $4 \nmid a$

d. If $6 \mid q$, then $3 \mid q$

e. If $9 \nmid a$, then $3 \nmid a$

f. If $6 \nmid q$, then $3 \nmid q$

DIVISIBILITY RULES

DIVISIBLE BY	TEST
A number is divisible by 2	**If the units digit is divisible by _____**
A number is divisible by 5	**If the units digit is a _____ or a _____**
A number is divisible by 10	**If the units digit is a _____**
A number is divisible by 3	**If the sum of the digits is divisible by _____**
A number is divisible by 9	**If the sum of the digits is divisible by _____**
A number is divisible by 4	**If the last two digits represent a number that is divisible by _____**
A number is divisible by 8	**If the last three digits represent a number that is divisible by _____**
A number is divisible by 6	**If the number is divisible by both _____ and _____**
A number is divisible by 15	**If the number is divisible by both _____ and _____**
A number is divisible by 11	**If the sum of the digits in the places that are even powers of 10 *minus* the sum of the digits in the places that are odd powers of 10 is divisible by _____**

Test the following numbers.
SHOW ALL WORK AND BE VERY CLEAR ABOUT THE "WHY" !!!!

IS 2,918,322 DIVISIBLE BY	WHY or WHY NOT?
2	
3	
4	
5	
6	
8	
9	
10	
11	
15	

IS 57,729,364,583 DIVISIBLE BY	WHY or WHY NOT?
2	
3	
4	
5	
6	
8	
9	
10	
11	
15	

The store manager has an invoice for 72 calculators. The first and last digits on the receipt are smeared. The manager can read $\$ \square 67.9 \square$. What are the missing digits? What is the cost per calculator?

If the number is divisible by 72, then the number is also divisible by everything that divides 72. List the factors of 72:

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____, and _____,

Knowing that 2 is on this list, the choices for the last digit could be: _____

Knowing that 4 is on this list, narrow the list for the last digit down to _____

Knowing that 8 is on this list, the last digit could only be _____

Knowing that 9 is on this list, the first digit has to be _____

A class visited a neighborhood cannery. The manager told the class that there were 11,368 cans of juice already boxed up in the inventory ready to be shipped out. He also said that the cans were packed in boxes of 6 or 24, depending on the size of the can. Billy spoke up and said that this cannot be so and there must be a mistake in the inventory. Is Billy right? Why or why not?