**T101 SECTION 5-2 MULTIPLICATION AND DIVISION OF INTEGERS**

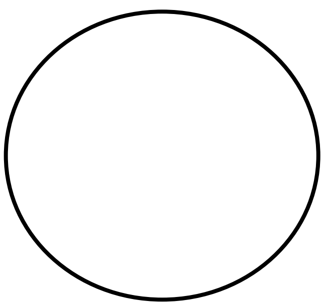
## I. MULTIPLICATION

### A. PATTERNS MODEL (Start with what we know and extend into the *unknown*)

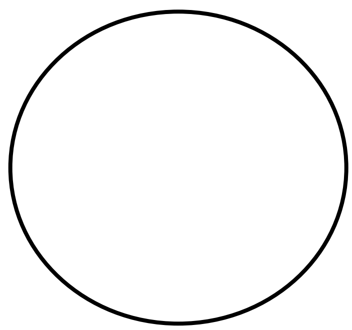
| **Positive times a Negative** | **Negative times a Negative** |
|---|---|
| 3 · 2 = 6 | 3 · -2 = ______ |
| 3 · 1 = 3 | 2 · -2 = ______ |
| 3 · 0 = ______ | 1 · -2 = ______ |
| 3 · -1 = ______ | 0 · -2 = ______ |
| 3 · -2 = ______ | -1 · -2 = ______ |
| 3 · -3 = ______ | -2 · -2 = ______ |

### B. CHARGED-FIELD MODEL (OR CHIP MODEL)

**First multiplicand:** Tells how many groups to put in (if +) or take out (if -)
**Second Multiplicand:** Tells which type of charges

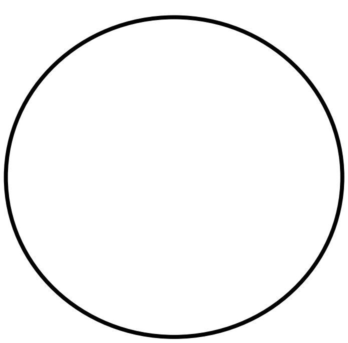


Show 3 · -2
Which means:
*Put in 3 groups of*
*2 negative charges*



Show –3 · -2
Which means:
*Take out 3 groups of*
*2 negative charges*
So start with a
"neutral zone"

(Notice the "repeated addition" -2 + -2 + -2)



Show –3 · 2
(Which means:
___________________
___________________
___________________)

C. **NUMBER-LINE MODEL**

Use a moving car, but under the following rules

1. Traveling west (left) is moving in the negative direction
2. Traveling east (right) is moving in the positive direction
3. Time in the future is a positive value
4. Time in the past is a negative value

So,

1. 30 x 2 (means start at zero and head east at 30 mph. Where are you in 2 hours from now?)

-90 -60 -30 0 30 60 90

2. -30 x 2 (means start at zero and head west at 30 mph. Where are you in 2 hours from now?)

-90 -60 -30 0 30 60 90

3. 30 x -2 (means start at zero and head east at 30 mph. Where were you 2 hours ago?)

-90 -60 -30 0 30 60 90

4. -30 x -2 (means start at zero and head west at 30 mph. Where were you 2 hours ago?)

-90 -60 -30 0 30 60 90

**D. PROPERTIES OF INTEGER MULTIPLICATION**

Given integers *a, b,* and *c:*

**A. Closure Property of Multiplication of Integers**

For any integers a and b, ________________________________________________

**B. Commutative Property of Multiplication of Integers**

For any integers a and b, _________________________________

**C. Associative Property of Multiplication of Integers**

For any integers a, b, and c, _______________________________________________

**D. Identity Property of Multiplication of Integers**

There is a unique integer ______, called the *multiplicative identity* such that for any integer a,

_________________________________________________

**E. Distributive Properties of Multiplication over Addition of Integers**

a (b + c) = __________________________

**F. Zero Multiplication Property of Integers**

For every integer *a,* ___________________________________________________________

**G. Others:**

For any integers, *a* and *b:*

1. (-1) · a = ______________________
2. (-a) · b = ______________________
3. (-a) · (-b) = _____________________

Using the Distributive Property, simplify each of the following so that there are no parentheses in the final answer.

(-2)( x + 5)     a ( x – 7)     (a + b)(a – b)     (-2 + b)(-2 – b)

From what you have learned above, that is, $(a + b)(a - b) = a^2 - b^2$, among others, let's go in the opposite order and FACTOR each of the following completely.

$x^2 - 16$     $-3x + 5xy$     $5x^2 - 7y^2$     $(x + y)^2 - z^2$

## II. INTEGER DIVISION

DEFINITION OF DIVISION

If a and b are any integers, with $b \neq 0$, then $a \div b$ is the unique integer c, if it exists, such that:

Use the definition of division (from above) to evaluate (if possible) the following:

$15 \div (-3)$ $-36 \div 9$ $-60 \div (-12)$ $-16 \div 3$ $-9 \div 0$

Change each of the following to a multiplication problem:

$82 \div b = 41$ $c \div 9 = -k$ $p \div m = 75$

## III. ORDER OF OPERATIONS (Please Excuse My Dear Aunt Sally)

$2 - 6 \times 3 + 5$ $(3 - 7) \cdot 8 + 2$ $3 - 4 \cdot 5 + 2 \cdot 5 - 1 + 5$

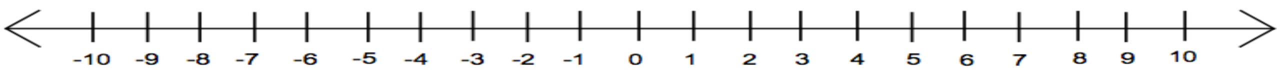
$2 + 16 \div 4 \cdot 2 + 8$ $(-2)^6$ $-2^6$

$(-3)^3$ $-3^3$

**DO: Order of Operations Worksheet**

## IV. ORDERING INTEGERS

As with whole numbers, a number line can be used to describe greater-than and less-than relations with integers.




Because -8 is to the __________ of -2 on the number line, we say that -8 is __________ than -2.

Because 3 is to the ___________ of -2 on the number line, we say that 3 is ______________ than -2.

## SOLVING INEQUALITIES | EXAMPLES

**If $x < y$ and n is any integer, then ______________________**
*You can ADD any integer to each side of the inequality and not change the "truth" of it. Since n can be any integer, this also means you can subtract the same integer from each side as well.*

$-4 < 6$ (add -3 to each side)

**If $x < y$, then ______________________**
*If you multiply both sides of an inequality by -1, it changes the "truth" of the inequality. Therefore, the inequality must be "flipped" or "reversed".*

$-4 < 6$ (times each side by -1)

**If $x < y$ and $n > 0$, then ______________________**
*If you multiply both sides of an inequality by a positive integer, it does not change the "truth" of the inequality. Therefore, the inequality **IS NOT** "flipped" or "reversed".*

$-4 < 6$ (times each side by 2)

**If $x < y$ and $n < 0$, then ______________________**
*If you multiply both sides of an inequality by a negative integer, it changes the "truth" of the inequality. Therefore, the inequality must be "flipped" or "reversed".*

$-4 < 6$ (times each side by -2)

Solve each of the following. **SHOW EVERY STEP AND USE A HORIZONTAL FORMAT**:

$4 + a > -5$ $3 - 2x + 3x < -2$ $-x - 5 \geq -13$