

In this chapter, we are going to begin with the integers and then develop an understanding of number theory (even, odd, prime, composite, divisibility, GCD, LCM, etc.).

Thus far, we have examined

1. Natural Numbers, $\mathbf{N} = \{ \underline{\hspace{10cm}} \}$

2. The Whole Numbers, $\mathbf{W} = \{ \underline{\hspace{10cm}} \}$

Now we will union the set of whole numbers (\mathbf{W}) with the set of **negative integers (the numbers to the left of zero)**. In doing so, we create:

3. The Integers, $\mathbf{I} = \{ \underline{\hspace{10cm}} \}$

NOTE: $\mathbf{N} \subset \mathbf{W} \subset \mathbf{I}$

SECTION 5-1 INTEGERS AND THE OPERATIONS OF ADDITION AND SUBTRACTION

I. REPRESENTATION

We have seen above that we are using the “-” symbol to indicate a negative number and to indicate subtraction. This symbol can also be thought of as “the opposite of”.

For instance, the opposite of 7 is -7 . Similarly, the opposite of -4 is 4.

So what is $-(-9)$? _____ What is the opposite of 0? _____

Note that $-x$ is not necessarily a negative number. Why?

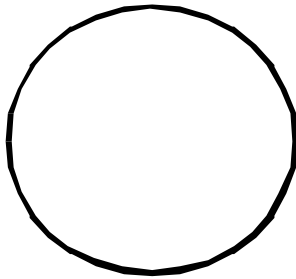
Example: Find $-x$ when $x = -5$

II. INTEGER ADDITION

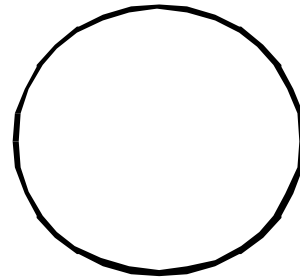
A. CHIP MODEL

In the chip model, positive integers are represented by black chips (•) and negative integers by red chips (•). One red chip *neutralizes* one black chip.

Show $-5 + 2$



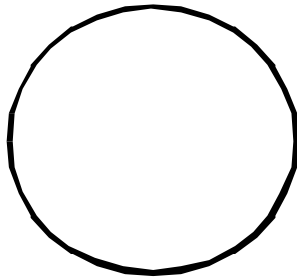
Show $-3 + -2$



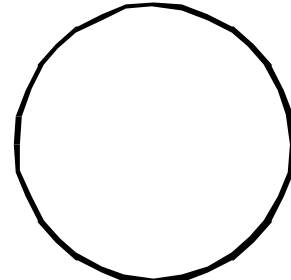
B. CHARGED FIELD MODEL

A model similar to the chip model that uses *positive and negative charges*.

Show $4 + -6$



Show $-9 + 3$



C. PATTERN MODEL

We extend the addition facts that we know from whole numbers into the negative integers. Keep one addend "fixed" and decrease the other by 1. Notice what happens to the sum.

$$3 + 4 = 7$$

$$3 + 3 = 6$$

$$3 + 2 = 5$$

$$3 + 1 =$$

$$3 + 0 =$$

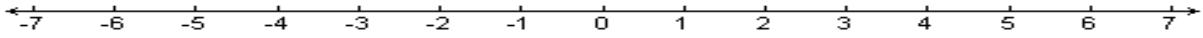
$$3 + -1 =$$

$$3 + -2 =$$

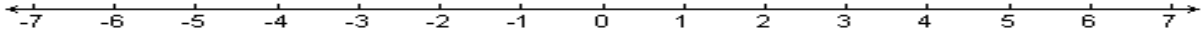
D. NUMBER LINE MODEL

Using a number line and a moving object (a car) we can represent positive integers with the car moving forward and negative integers with the car moving in reverse.

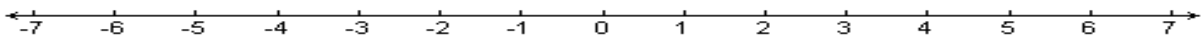
Show $4 + -3$



Show $-3 + -2$

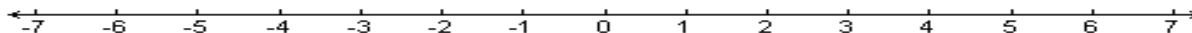


Show $-5 + 7$



III. ABSOLUTE VALUE

Because -6 and 6 are opposites, they are on opposite sides of 0 on the number line and they are the same distance (_____ units) from 0 .



The distance between an integer, x , and zero on the number line is also known as *the absolute value of x* and is denoted _____.

Notice that $|-6| = \underline{\hspace{2cm}}$ and $|6| = \underline{\hspace{2cm}}$.

Since absolute value implies “distance”, it is always positive.

Definition of Absolute Value

$$|x| = \hspace{10em} \text{If } x \geq 0$$

$$|x| = \hspace{10em} \text{If } x < 0$$

(Since x is negative to start, the $-(x)$ will be a positive value in the end)

So, $|32| = \hspace{2em}$ $|-17| = \hspace{2em}$ $-|-9| = \hspace{2em}$ $|2-7| = \hspace{2em}$

IV. PROPERTIES OF INTEGER ADDITION

Given integers a , b , and c :

A. Closure Property of Addition of Integers

For any integers a and b , _____

B. Commutative Property of Addition of Integers

For any integers a and b , _____

C. Associative Property of Addition of Integers

For any integers a , b , and c , _____

D. Identity Property of Addition of Integers

There is a unique integer _____, called the *additive identity* such that for any integer a , _____

E. Uniqueness Property of the Additive Inverse

For every integer a , there exists a unique integer $(-a)$, called the additive inverse of a , such that: _____

F. Properties of the Additive Inverse

For any integers, a and b :

1. $-(-a) =$ _____

2. $-a + -b =$ _____

V. INTEGER SUBTRACTION

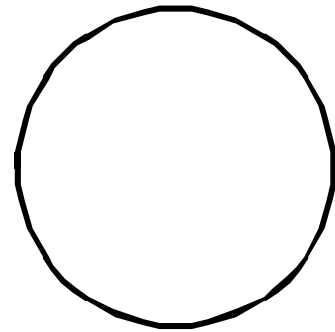
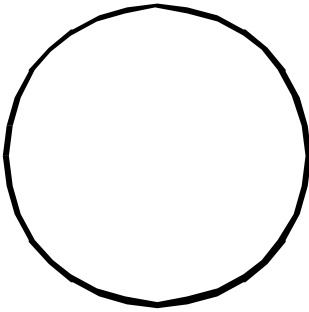
A. CHIP MODEL

We need to put in enough black (positive ●) or red (negative ●) chips to represent the first number and enough to have a “neutral zone” to subtract from.

Show $5 - -2$

Think: “5 ●’s take away 2 ●’s”

Show $-3 - -4$



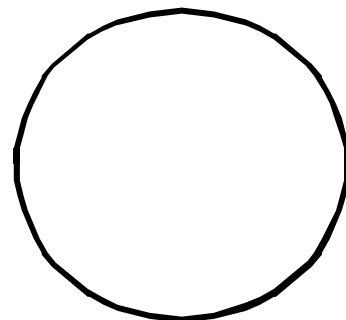
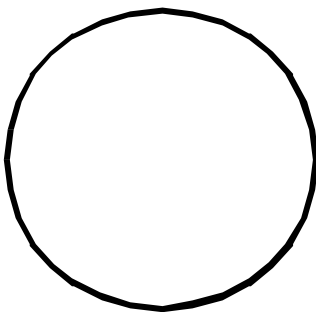
B. CHARGED-FIELD MODEL

Again, we need to represent the first number with either + or - signs and then have a “neutral zone” from which we can subtract.

Show $-3 - 5$

Think: “3 -’s take away 5 +’s”

Show $7 - -4$



C. **PATTERNS MODEL**

We extend the subtraction facts that we know from whole numbers into the negative integers. Keep the minuend "fixed" and increase or decrease the subtrahend by 1. Notice what happens to the difference.

$$3 - 2 =$$

$$3 - 2 =$$

$$3 - 1 =$$

$$3 - 3 =$$

$$3 - 0 =$$

$$3 - 4 =$$

$$3 - ^{-}1 =$$

$$3 - 5 =$$

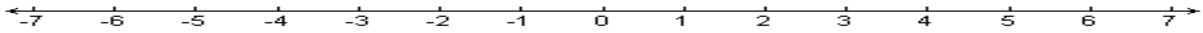
$$3 - ^{-}2 =$$

$$3 - ^{-}3 =$$

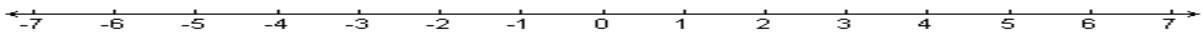
D. NUMBER LINE MODEL

The car starts at zero and is pointed in a positive direction (to the right). Subtraction corresponds to facing the car in a negative direction. We subtract a positive integer by moving the car forward and a negative integer by moving the car in reverse.

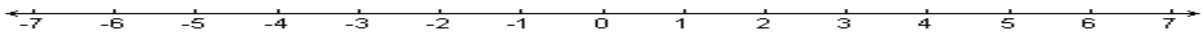
Show $4 - 3$



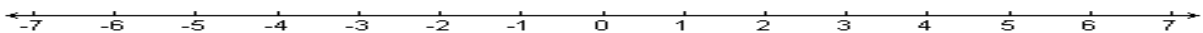
Show $4 - ^{-}3$



Show $^{-}4 - ^{-}3$



Show $^{-}4 - 3$



VI. SUBTRACTION USING THE MISSING ADDEND MODEL

Subtraction of integers, like subtraction of whole numbers, can be defined in terms of addition.

Recall, $5 - 3 = n$ if and only if $5 = 3 + n$

Similarly, $3 - 5 = n$ if and only if _____

DEFINITION OF SUBTRACTION

For integers a and b , $a - b$ is the unique integer n such that:

Rewrite as an Addition Problem. DO NOT LEAVE DOUBLE SIGNS

$$4 - 7 = -11$$

$$b - 3 = -6$$

$$k - y = w$$

VII. SUBTRACTION USING ADDING THE OPPOSITE APPROACH

We know that: $12 - 5 = \underline{\hspace{2cm}}$

We also know that: $12 + \overset{-}{5} = \underline{\hspace{2cm}}$

PROPERTY

For integers a and b , $a - b = \underline{\hspace{2cm}}$

Use the fact that $a - b = a + \overset{-}{b}$ to compute the following:

$$6 - 11 = \underline{\hspace{2cm}} \qquad -4 - 12 = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \qquad = \underline{\hspace{2cm}}$$

$$\overset{-}{12} - \overset{-}{5} = \underline{\hspace{2cm}} \qquad 8 - \overset{-}{11} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \qquad = \underline{\hspace{2cm}}$$

Simplify each of the following:

$$\overset{-}{(x - y)}$$

$$-5 - (x - 6)$$

$$\overset{-}{(x - y)} - y$$

VIII. ORDER OF OPERATIONS

“Please Excuse My Dear Aunt Sally”

1. Do any computation inside **P**arentheses FIRST – or any grouping symbol.

$$(a+b) \quad [a+b] \quad \{a+b\} \quad |a+b| \quad \frac{a}{b}$$

2. If there are any **E**xponents, do those next.
3. Do all **M**ULTIPLICATIONS and **D**IVISIONS as they occur from LEFT to RIGHT
4. Do all **A**DDITIONS and **S**UBTRACTIONS as they occur from LEFT to RIGHT

EXAMPLES: SHOW EACH & EVERY STEP

$$2 - 5 - 7$$

$$2 - 3 \cdot 4$$

$$2^2 + 3 \cdot 2 - 1$$

$$3 - (7 - 3) \div 2$$