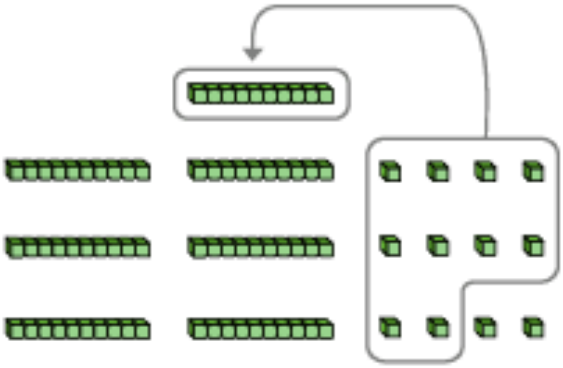
T101 SECTION 3-4 ALGORITHMS FOR WHOLE NUMBER MULTIPLICATION AND DIVISION

I. MULTIPLICATION ALGORITHMS

Recall that we define multiplication as repeated addition. So, 24 x 3 means 24 + 24 + 24

A. Use Base-10 Blocks to multiply (the concrete representation):



$$\begin{array}{r} {\scriptstyle 1} \\ 24 \\ \times\ 3 \\ \hline 72 \end{array}$$

This then evolves into the algorithms (the abstract representation):

Tens	Ones
2	4
x	3

(20 x 3) + (4 x 3) =

$$\begin{array}{r} 20 + 4 \\ \times\quad 3 \\ \hline + \end{array}$$

Standard Algorithm

$$\begin{array}{r} 24 \\ \times\quad 3 \\ \hline \end{array}$$

B. Multiplication By Powers of 10 (10^n)

Let's examine 34 x 10 using the Base-10 blocks.

- Start by representing 34
- Then multiply the 4 units by 10. (4 x 10 = 40)
- Replace the 4 units with 4 longs
- Then multiply the 3 longs by 10. (30 x 10 = 300)
- Replace the 3 longs with 3 flats
- What number is now represented by the blocks? _____
- Thus 34 x 10 = _____

In general, when multiplying any natural number by 10^n, where n is a natural number, the result is simply the annexing *of n* zeroes to the number. *(This is also true in any other base n system!)*

EXAMPLES:

28×10^5 = _____ 53×10^3 = _____

$23_{\text{five}} \times 10_{\text{five}}$ = _____ $101_{\text{two}} \times 100_{\text{two}}$ = _____

C. Product Rule for Exponents

Now consider $10^4 \times 10^3 =$ _____ x _____ = _____

If a is a natural number and m and n are whole numbers, then $a^m \times a^n =$ _____

$2^4 \times 2^3 \times 2^5 =$ $5^3 \times 5^5 =$ $2^3 \times 3^2 =$

D. Multiplication with Two-Digit Factors

Standard Algorithm

```
  23
x 14
```

```
  23
x 14
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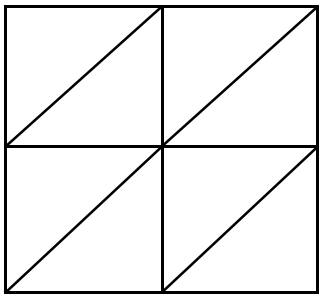
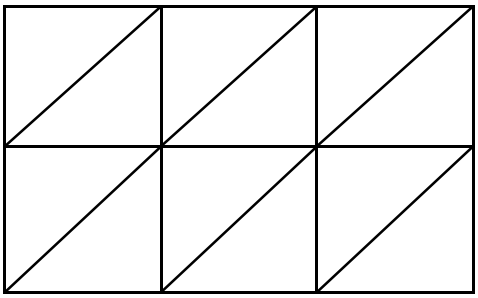
(3 x 4)
(20 x 4)
(3 x 10)
(20 x 10)

E. Lattice Algorithm for Multiplication

Multiply 23 x 14



Now, you try: multiply 348 x 25



II. DIVISION ALGORITHMS

A. Repeated Subtraction (*Turbo Style*) to Develop the Standard Algorithm

We are packing *Starburst* candies into packs that hold 5 candies each. If we have 629 candies, how many packs can we make?

Note:
If 1 pack holds 5 candies

then 10 packs hold _____ candies

then 100 packs hold _____ candies

$5\overline{)629}$

B. Scaffolding Technique

$4\overline{)531}$

C. Standard Algorithm

$7\overline{)1029}$

D. Using Base-10 Blocks to Develop the Standard Algorithm

Let's model the problem 725 ÷ 6

- Represent 725 with the Blocks
- Group the largest piece in sets of 6 (the divisor)
- Convert any leftover pieces into the next block down
- Group the next largest piece in sets of 6
- Continue ...
- To determine the answer, determine how many groups of each piece

E. Division by Two-Digit Divisor

CONVENTIONAL TECHNIQUE

Most elementary texts have the students divide using 4-Step algorithm:
Estimate, Multiply, Subtract, Compare

$32\overline{)2,618}$

SCAFFOLD TECHNIQUE

$31\overline{)3,458}$

Now use the ***division algorithm*** to check your solution. 2,618 = _____ x _____ + _____

F. Short Division

The last few examples demonstrate **long division**. When the divisor is a one-digit number, you can use **short division.**

$5\overline{)2,880}$

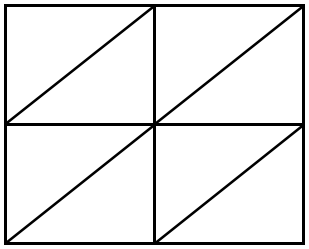
III. MULTIPLICATION AND DIVISION IN DIFFERENT BASES

Conventional Method

$$\begin{array}{r} 3\,2_{\text{five}} \\ \times\quad 4_{\text{five}} \\ \hline \end{array}$$

Lattice Multiplication

$43_{\text{five}} \times 24_{\text{five}}$



Now let's try a division problem.

1. First, we'll use *repeated subtraction. along with scaffolding.*

$$23_{\text{five}} \overline{)\ 4\ 1\ 0\ 3_{\text{five}}}$$

2. Now let's use the more *conventional method.*
(You'll need to know your multiplication facts!)

$$13_{\text{five}} \overline{)\ 2\ \ 2\ \ 3\ 4_{\text{five}}}$$

TAKE HOME PROBLEMS

Draw a representation of division using Base 10 blocks

$533 \div 4$

Use Short Division:

$8\overline{)34,951}$

Use the Conventional Method

24_{five}

$\times 33_{five}$

Use Lattice Multiplication

$121_{three} \times 22_{three}$

Divide using any Long Division Algorithm (use Base 2)

$110\,101_{two} \div 11_{two}$