T101 SECTION 2.2 DESCRIBING SETS

Introduction to Set Theory

A. Set: A collection of objects

Example:

The letters in the word **tall** can be written using set notation as:

T = or or

The order in which the elements are written makes no difference, and ***each element is listed only once.***
The letters in the word **usually** can be written using set notation as:

A = {

B. Elements or Members: the individual objects in a set. The symbol $\in$ means "is an element of" and the symbol $\notin$ means "is not an element of".
True or False: **t ∈ T** **a ∈ T** **t ∉ A** **{a} ∈ T** **{s} ∉ T**

C. Well-Defined Sets: The definition of the set is clear enough that we are certain what is or is not an element

Example of a well-defined set:

Example of a not well-defined set:

D. Set-Builder Notation: used when the elements of a set are quite numerous (or infinite)

Example: The set of natural numbers between 0 and 100.

Write the following using set-builder notation:
1. The set of states in the United States:

2. {2, 4, 6, 8, ...}

E. Listing Method: when each element of a set is listed
Example: The set of numbers between 2 and 9.

Write the following using the listing method: R = {2r+1 | r = 3, 4, 5}

F. **Cardinal Number**: the number of elements in a set **A**, denoted: _____.

Example: If **R** = {1, 2, 3, 4} and **S** = {a, b, c, d}, then

n(**R**) = _____ and n(**S**) = _____.

G. **Equal Sets**: Two sets that contain exactly the same elements (order does not matter)

Example: If **A** = {t, o, p} and **B** = {p, o, t}, then _____.

H. **Equivalent Sets**: Two sets that have the same cardinal number.
From above, **R** and **S** are equivalent. **R ~ S** is the notation.

Examples: **A** = {p, q, r, s} **B** = {a, b, c} **C** = {x, y, z} **D** = {b, a, c}
Compare with = and **~** :

A & B_____ B & C _____

C & D_____ B & D _____

I. **Empty (or Null) Set**: the set with no elements, denoted

_____ or _____ but NOT _____.

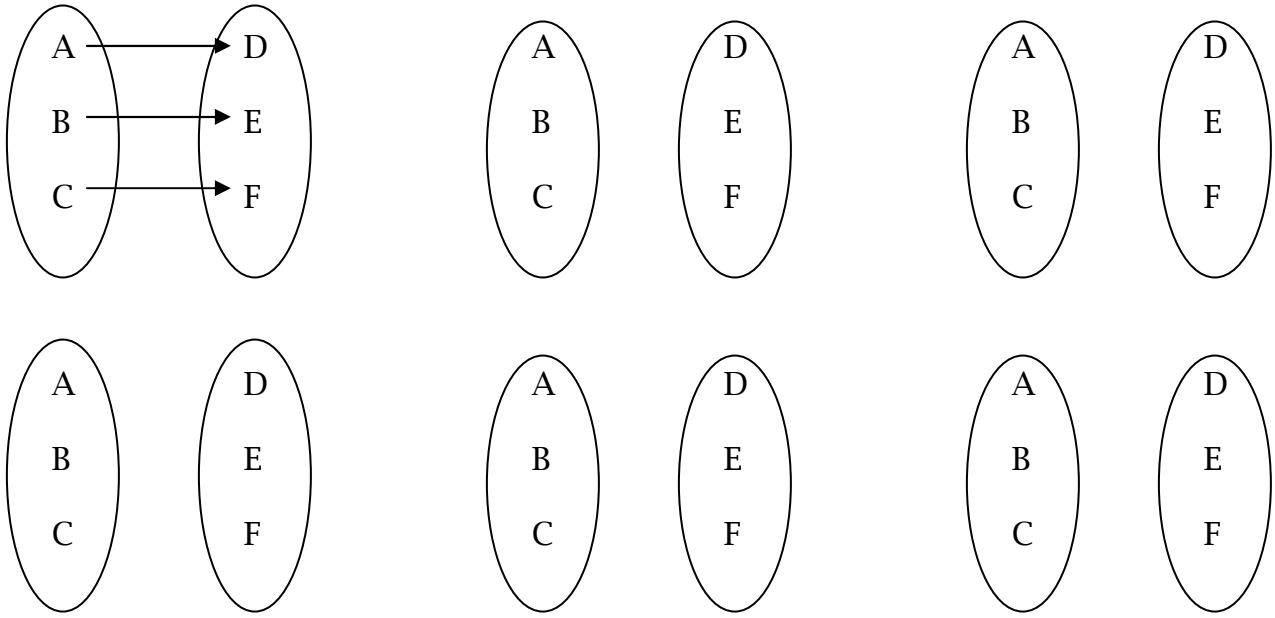
J. **Finite Set**: A set whose cardinal number is zero or a natural number (any whole number).
Example:

K. **Infinite Set**: A set that is not finite.
Example:

L. One-to-One Correspondence: A one-to-one correspondence exists between two sets (say, M and N) if the elements of M and N can be paired so that for each element of M there is exactly one element in N ***and*** for each element of N there is exactly one element in M.

Given G = {Ann, Betty, Cindy} and B = {Dan, Ed, Frank}, is it possible to set up a one-to-one correspondence between these two sets?

Set up all possible one-to-one correspondences.
(SET & ARROW METHOD)



Given A = {a, b} and B = {1, 2, 3}, is it possible to set up a one-to-one correspondence between these two sets?

How many ways can the colors red, white, and blue be arranged on a flag with 3 stripes? (ie. How many one-to-one correspondences exist?)
(TREE DIAGRAM METHOD)

M. Fundamental Counting Principle: If event **M** can occur in ***m*** ways and, after it has occurred, event **N** can occur in ***n*** ways, then event **M** followed by event **N** can occur in (***m x n***) ways. (We use this to **"count"** different arrangements or correspondences)

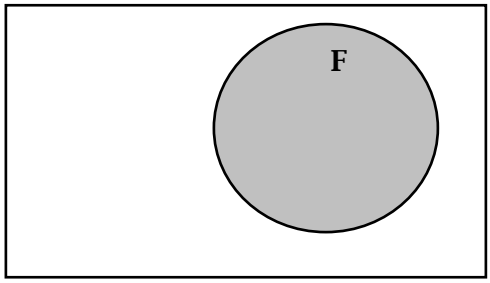
How many ways can the colors red, white and blue be arranged on a flag with 3 stripes?

# Choices for top stripe		# Choices for middle stripe		# Choices for bottom stripe
	x		x	=

N. Universal Set: Denoted U, it is the set that contains all elements being considered in a particular discussion. Note that **U** will change from problem to problem.

Example: Let **U** = {*x* | *x* is a student at IUS} and
F = {*x* | *x* is a female student at IUS}

Venn Diagram:

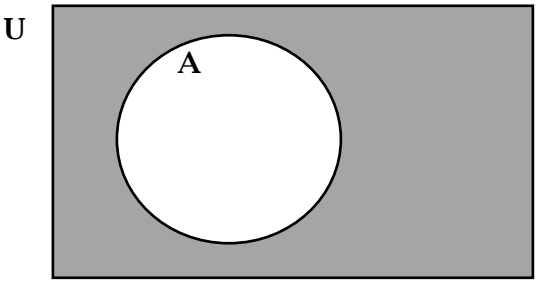


O. Complement of a Set A: the set of all elements in the universal set **U** that are not in **A**, denoted _____. (Said as "not A" or "complement of A")

Example: In the above Venn diagram, what would you call the elements in **U** that are not in **F**?

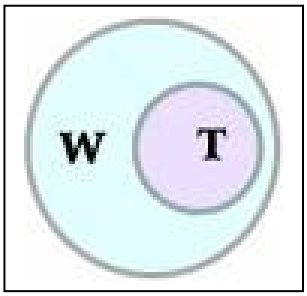
Example: If **U** = {u, n, i, v, e, r, s, a, l} and **A** = {s, e, a, l}, what is $\overline{A}$ **?**

Venn Diagram



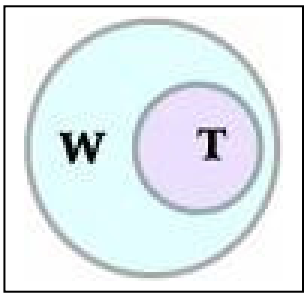
P. **Subset:** **T** is a subset of **W**, denoted_____, if and only if *every* element in **T** is an element of **W**. *(Note that this definition allows **T** to be equal to **W.**)*

Venn Diagram: **U**



Q. **Proper Subset:** If **T** is a subset of **W** and **T** is not equal to **W**, *then* **T** is a *proper subset* of **W**, denoted _____. *(Note that this definition does not allow **T** to be equal to **T.** Thus every element in **B** must be in **A** and there must be at least one element of **W** that is not in **T.**)*

Venn Diagram: **U**



EXAMPLE: If **K** = {k, i, t}, list all the **subsets** of **K.**

EXAMPLE: If **K** = {k, i, t}, list all the **proper subsets** of **K.**

How many subsets did a set with 3 elements have? _____

How many proper subsets did a set with 3 elements have? _____

? IS THE EMPTY SET IS A SUBSET OF EVERY SET?

? IS THE EMPTY SET IS A PROPER SUBSET OF EVERY SET?

? IS EVERY SET IS A SUBSET OF ITSELF?

? IS EVERY SET IS A PROPER SUBSET ITSELF?

R. Number of Subsets of a Set

Let's try an experiment that will help us know how many subsets any given set will have.

Set	Number of Elements in the Set	List all Subsets of the Set	Number of Subsets	Number of Proper Subsets	
∅ or { }	0				
{a}	1				
{a,b}	2				
{a,b,c}	3				
{a,b,c,d}	4	{a,b,c,d} {a,b,c} {a,b,d} {a,c,d} {b,c,d} {a,b} {a,c} {a,d} {b,c} {b,d} {c,d} {a} {b} {c} {d} { }			
{a,b,c,d,e}	5				
{a,b,c,d,e,f,g,h,i,j,k}	11				
{a,b,c…}	n				

CONCLUSION: A set with n elements will have _____subsets and _____ proper subsets.

TAKE HOME PROBLEMS:

Example: If **A** = {x|x is a letter in the word *band* }
B = {x|x is a letter in the word *banana* }
C = {x|x is a letter in the word *bandana* }

What is **n(A)**? What is **n(B)**? What is **n(C)**?

Is **B** $\subseteq$ **A**? Is **B** $\subset$ **A**? Is **C** $\subseteq$ **A**? Is **C** $\subset$ **A**?

Is **A** $\subseteq$ **B**? Is **A=B**? Is **A=C**?