

M110 SECTION 5.1 THE EVOLUTION OF NUMERATION SYSTEMS

Do you know what all the following have in common?



1001_{two}

14_{five}



IX

What is a numeration system? *A set of numerals and a method of arranging the numerals to represent numbers.*

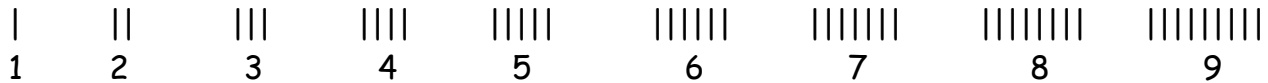
What is the name of our numeration system?

What is the difference between a number and a numeral?

EGYPTIAN NUMERATION SYSTEM (1500-1600 BC)

The Egyptian hieroglyphic system is an example of a *simple grouping* numeration system. The numbers are formed by combining copies of the symbols (hieroglyphics) that represent the powers of ten.

This numeration system is based upon tally marks. That is,



But after 9, they needed a new symbol that began "grouping" to represent certain *sets* of numbers. The next value was ∩ (heel bone) that represented 10.

Egyptian Numeral	Description	Hindu-Arabic Equivalent
	Stroke	1
∩	Heel Bone	10
	Scroll	100
	Lotus Flower	1,000
	Pointing Finger	10,000
	Fish or Tadpole	100,000
	Astonished person	1,000,000

The Egyptian system simply followed an *additive property*; that is, the value of the number was the sum of the face values of the numerals.

Convert to Hindu-Arabic:



Write using Egyptian Numerals: 304,536

ADDING IN THE EGYPTIAN SYSTEM

We add by grouping all the symbols together. After obtaining the sum, we regroup any symbol that is repeated 10 or more times to the next higher symbol. This is similar to the "carrying" that we do in our Hindu-Arabic numeration system.

$$\begin{array}{r}
 723 \\
 + 593 \\
 \hline
 1316
 \end{array}$$

$$\begin{array}{r}
 \text{e e e e e e e e} \quad \text{n n} \quad \text{III} \\
 + \quad \text{e e e e e} \quad \text{n n n n n n n n n n} \quad \text{III} \\
 \hline
 \text{e e e e e e e e e e e e e e} \quad \text{n n n n n n n n n n n n n n} \quad \text{IIIIII} \\
 \text{which is simplified to} \quad \text{f e e e n IIIII}
 \end{array}$$

You try:

$$\begin{array}{r}
 \text{e e e} \quad \text{n n n n n n n n} \quad \text{IIIIII} \\
 + \quad \text{e e e e e e e e e e} \quad \text{n n n n} \quad \text{III} \\
 \hline
 \end{array}$$

SUBTRACTING IN THE EGYPTIAN SYSTEM

Similar to our Hindu-Arabic system, we may need to "borrow" from a higher symbol.

$$\begin{array}{r}
 \text{f f f f f} \quad \text{n n n n} \quad \text{II} \\
 - \quad \text{f f} \quad \text{e e e e} \quad \text{n n n} \quad \text{IIIIIIII} \\
 \hline
 \end{array}$$

First rewrite the problem aligning like symbols or numerals - give yourself some space!

$$\begin{array}{r}
 \text{f f f f f} \quad \quad \quad \text{n n n n} \quad \quad \text{II} \\
 - \quad \text{f f} \quad \quad \quad \text{e e e e} \quad \quad \text{n n n} \quad \quad \text{IIIIIIII} \\
 \hline
 \end{array}$$

ROMAN NUMERATION SYSTEM

(USED 500BC - 100AD - DURING THE REIGN OF THE ROMAN EMPIRE)

The Roman numeration system is a more sophisticated simple grouping system.

Advantages over the Egyptian system include:

- Utilizes a "subtraction principle" that allows numbers to be represented more concisely.
- Utilizes a "multiplication principle" that makes writing large numbers easier.

RULES:

- Only the symbols I, X, C, and M may be repeated, but never write a symbol more than 3 times in succession. If a fourth symbols seems necessary, use the subtraction principle.
- When using the subtraction principle, we can only subtract the symbols: I, X, C, and M (not V, L, or D - nothing with a "5")
- We can only subtract numerals from the next 2 higher numerals. (ex. you can have IV and IX, but you cannot have IL, IC, ID, IM)
- Use a bar above a symbol(s) to indicate multiplication by 1000
 \overline{V} means $5 \times 1000 = 5000$ \overline{IX} means $9 \times 1000 = 9000$
- Use vertical bars to indicate multiplication by 100
 $|V|$ means $5 \times 100 = 500$ $|L|$ means $50 \times 1000 \times 100 = 5,000,000$

Roman Numeral	Hindu-Arabic
	1
	5
	10
	50
	100
	500
	1000

EXAMPLES

If the Roman numerals are listed so that each numeral has a larger value than the numeral to its right, then the value of the numeral is found using an **Additive property**. Each of the numeral **I**, **X**, **C** and **M** may be repeated up to three times. The numerals **V**, **L**, and **D** are not repeated.

XVI =

CCCVI =

MMCCCLXII =

If the numerals are listed so that each numeral DOES NOT have a larger value than the numeral to its right, then the value of the numeral is found using an Additive property and a Subtractive property. The only numerals that may be subtracted from other numerals are I, X, and C.

IV =

IX =

XL =

XC =

CD =

CM =

CXLIV =

MCDLXXI =

Also, the subtraction of these values is allowed only if the value of the numeral to the right is within two rows on the table above of the numeral to the left.

XL =

XC =

but XD does not = 490 because X is 3 rows from D in the chart. 490 = _____

The Roman Numeration System also employs a Multiplicative property to write very large numbers.

$\overline{\text{IX}}$ =

$\overline{\text{DCCLXII}}$ =

$\overline{\text{V}}$ =

You try:

Convert to Base 10:

DCIV

MCDXLV

CMDXLVII

$\overline{\text{X}}$ MMCDLIV

Write using Roman Numerals:

579

4,709

304,536

8,070

CHINESE NUMERATION SYSTEM

The following symbols were used during the Han dynasty (206 B.C. - 220 A.D.).

Traditionally, the ancient Chinese wrote numerals vertically, but now they write them horizontally.

YOU WILL NOT NEED TO MEMORIZE THE CHINESE NUMERALS.

This system is a *multiplicative system*, using symbols for 1-9 and powers of 10.

$$\begin{array}{rcl} \text{五十七} & \text{means} & (5 \times 10) \\ & & 7 \\ & & = + \frac{7}{57} \end{array} = 50$$

1	一
2	二
3	三
4	四
5	五
6	六
7	七
8	八
9	九
10	十
100	百
1000	千

Express each of the following in Hindu-Arabic Notation:

七百四十三

二千四百七十九

Express each of the following using Chinese Notation:

842

7,609