M110 SECTION 15.4 THE NORMAL DISTRIBUTION

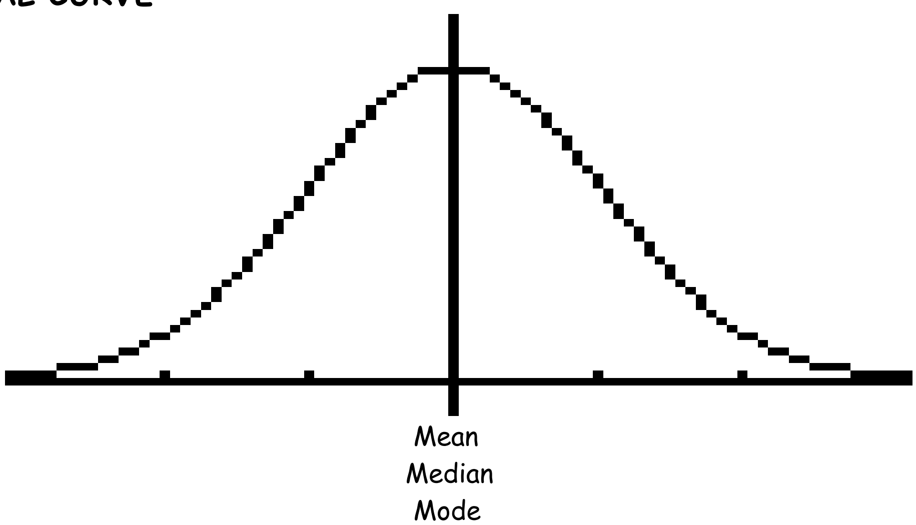
How do manufacturers decide how wide the seats should be at a movie theater?
How is a car designed so that most people do not bump their head on its interior roof?

Scientists gather data to answer questions such as these to ensure that MOST people will fit comfortable into their environments. Much of the work of these scientists is based on a curve that will study in this section called the ____

or the ____.

A **normal curve** is a smooth bell-shaped curve that depicts the frequency of the data values symmetrically about the mean.

THE NORMAL CURVE



Some properties of the Normal Curve:

1. The curve is bell-shaped.
2. The highest point on the curve is at the mean of the distribution.
3. The mean, median, and mode are all the same value.
4. The curve is symmetric with respect to the mean.
5. The total area under the curve is equal to 1.

When discussing normal distributions, we usually assume that we are dealing with an **entire population rather than a sample.** When this is the case we have new symbols for the mean and standard deviation:

Sample	Population	
		Mean
		Standard Deviation

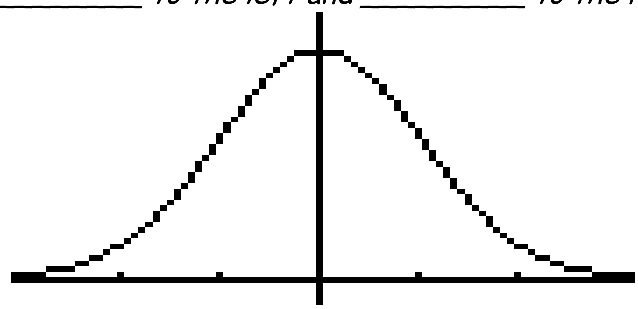
The normal curve has many uses. One is to calculate the percentage of data that lie above, below, or between particular data values. One rule to help us is as follows:

THE "68-95-99.7" RULE

Given a normal distribution with a mean, μ, and standard deviation, σ,

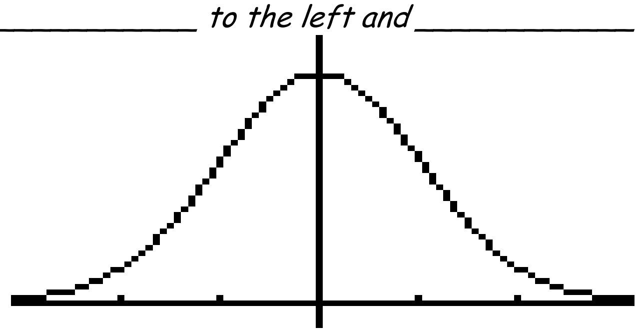
a. Approximately **68%** of the data lie *within* one standard deviation of the mean.

This means __________ *to the left and* __________ *to the right of the mean.*



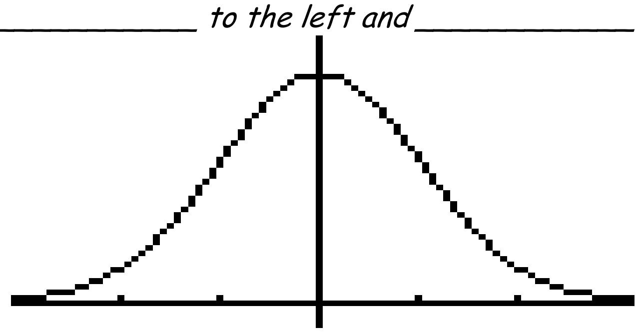
b. Approximately **95%** of the data lie *within* two standard deviations of the mean.

This means _____________ *to the left and* _____________ *to the right of the mean.*



c. Approximately **99.7%** of the data lie *within* three standard deviations of the mean.

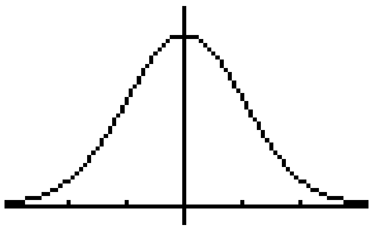
This means _____________ *to the left and* _____________ *to the right of the mean.*



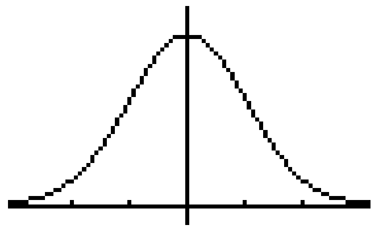
We can use the **68-95-99.7% Rule** to estimate ***how many values*** we expect to fall with one, two, or three standard deviations of the mean of a normal distribution.

EXAMPLE A distribution of scores of 1,000 students who take an IQ test is normally distributed with a mean of 450 and a standard deviation of 25.

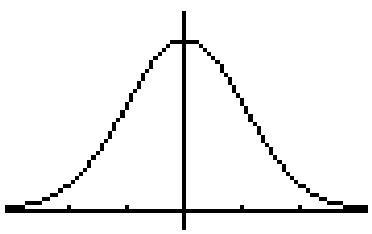
a. How many student scores do we expect to fall above 450?



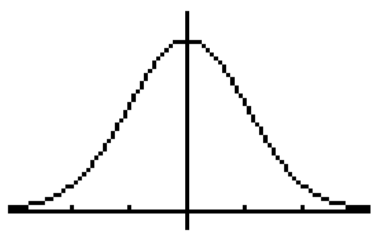
b. How many student scores do we expect to fall between 425 and 475?



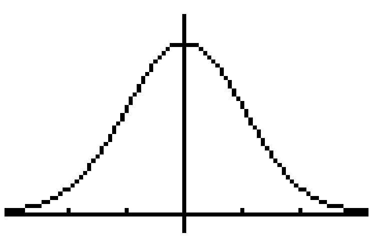
c. How many student scores do we expect to fall between 400 and 500?



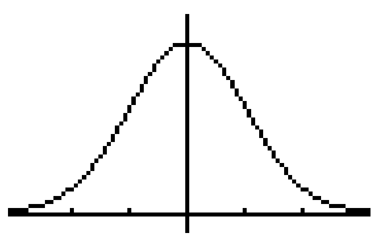
d. How many student scores do we expect to fall above 500?



e. How many student scores do we expect to fall below 425?



f. How many student scores do we expect to fall below 475?



But, what about areas under the curve that don't fit into the 68-95-99.7% Rule? That is, can we predict how many student scores we would expect to fall below 410?

It is possible to do so using:

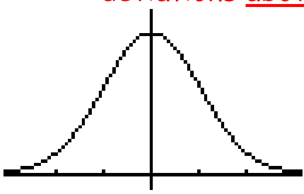
a. **z-score** (Tells the number of standard deviations that a raw score is from the mean.)
b. **A table of areas** under the standard normal curve. (A normal curve with a mean of 0 and a standard deviation of 1.)

FINDING AREAS UNDER THE STANDARD NORMAL CURVE

Given a standard normal distribution, find the percentage of data that lie in the following regions:

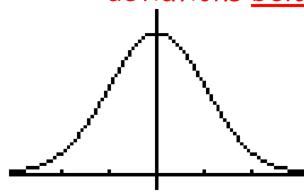
a. Between z = 0 and z = 1.3

(A z-score of 1.3 means 1.3 standard deviations above the mean.)

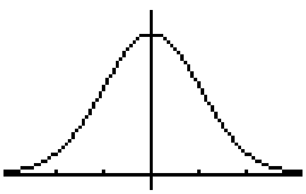


b. Between z = 0 and z = -1.83

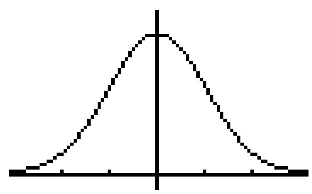
(A z-score of -1.83 means 1.83 standard deviations below the mean.)



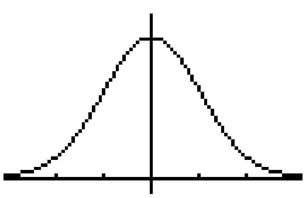
c. Between z = 1.5 and z = 2.1



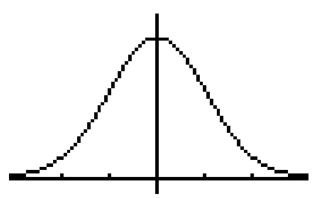
d. Between z = -.55 and z = -2.13



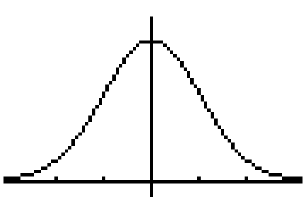
e. Above z = 1.45



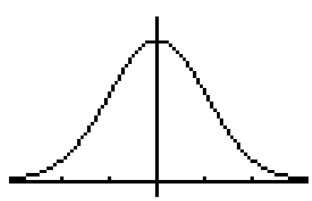
f. Below z = 1.22



g. Below z = -1.40



h. Above z = -0.46



But, real life data does not usually come in a standardized form with a mean of 0 and standard deviation of 1. A real life normal distribution, such as the set of all weights of women, 18-25 years old, may have a mean of 120 pounds and a standard deviation of 15 pounds. But we can still use the table if we first **CONVERT each RAW SCORE (Weight) to a z-SCORE.**

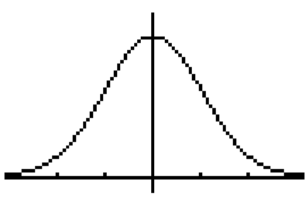
Z-SCORE FORMULA

z =

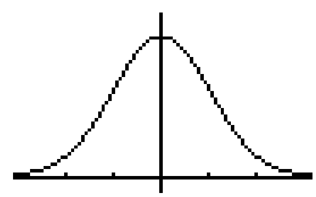
Round to 2 decimal places.

Convert each of the following raw (weight) scores to a z-score. The weights come from a population with a mean of 120 pounds and a standard deviation of 15 pounds. Then find the indicated percentage of data that lie in that region.

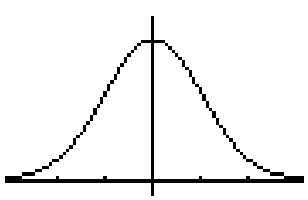
Above a weight of 117 pounds



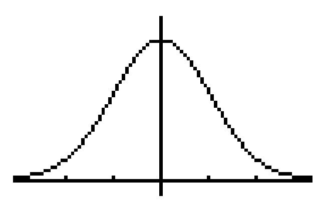
Below a weight of 125 pounds



A weight between 115 and 139 pounds



A weight between 122 and 133 pounds



Convert each of the following z-scores to a raw (weight) score. The weights come from a population with a mean of 120 pounds and a standard deviation of 15 pounds.

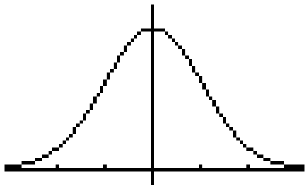
z = .84

z = -1.75

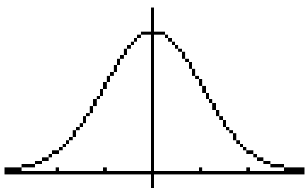
z = 2.03

USING AREA UNDER A STANDARD NORMAL CURVE TO FIND z-SCORES

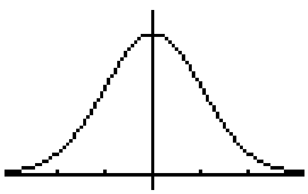
Find a z-score such that 10% of the area under the standard normal curve is above that score.



Find a z-score such that 12% of the area under the standard normal curve is below that score.



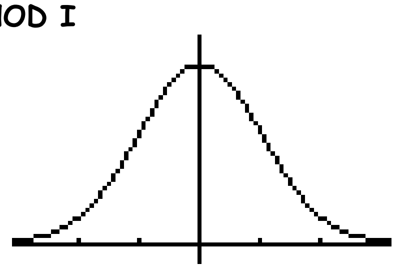
Find a z-score such that 70% of the area under the standard normal curve is below that score.



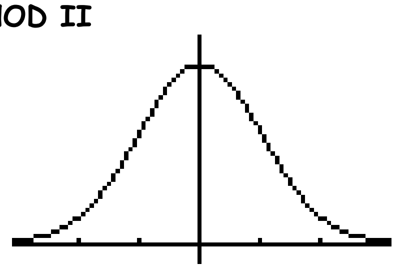
FINDING AREAS UNDER THE NON-STANDARD NORMAL CURVE

Suppose that to qualify for a management training program you must score in the top 10% of those employees who take an entrance exam. The scores are normally distributed with a mean of 65 and a standard deviation of 4. If you scored a 72, did you qualify for the training program?

METHOD I



METHOD II



FINDING AREAS/PERCENTAGES UNDER THE NON-STANDARD NORMAL CURVE

Applications of the normal distribution that are not standard, meaning the mean is not 0 and the standard deviation is not 1 -- more realistic cases. By using a simple conversion, we can standardize any normal distribution.

Step 1: Draw graph shading the area desired.

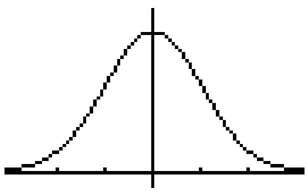
Step 2: Find z-score which shifts to the standard normal distribution where $\bar{x} = 0$, $s = 1$

Step 3: Use Table to find areas/percentages.

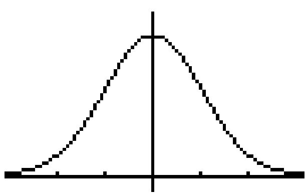
Step 4: Answer problem.

Assume that IQ scores are normally distributed with a mean of 100 and standard deviation of 15.

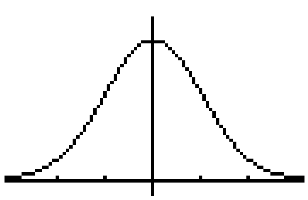
a. Find the percentage of adults that have an IQ greater than 131.5 (the requirement for membership in the Mensa organization)



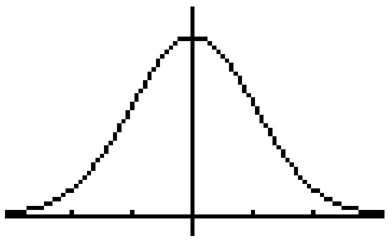
b. Find the percentage of adults that have an IQ between 80 and 95.



c. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days. One classical use of the normal distribution is inspired by a letter to "Dear Abby" in which a wife claimed to have given birth 308 days after a brief visit from her husband, who was serving in the Navy. Given this information, find the probability (which is the probability) that a pregnancy lasts 308 days or longer. What does the result suggest?



Men's resting heart rates are normally distributed with a mean of 68 beats per minute and standard deviation of 4 beats per minute. If 200 men are examined, how many would you expect to have a heart rate of less than 70?



Less than 65?