

Lesson 8.3 (F'11)

- Objective:
1. To compute simple interest.
 2. To compute compound interest.

An ad for a bank says that a Money Market account yields 2.65% interest. What does this mean?

Terms

Interest-is the amount of money the bank will pay for using your money or
-is the extra money you pay back to the bank when you borrow their money

Principal-the original amount deposited or borrowed.

Interest rate-the percent used to determine the interest

Simple interest-interest computed on the original principal

Example-If you borrow \$800 from a bank at a **simple interest** rate of 7% for one year, then the interest is_____

Let's do this without a formula

The amount needed to pay off the loan is_____

*Simple Interest after one year $I = Pr$

Simple Interest after t years $I = Prt$

It is VERY important to note that **time**, t , is always expressed in the **same period as the rate**.

If you are given the **annual** interest rate, then time must be in **years**.

If you are given a **monthly** interest rate, the time should be in **months**.

Interest rates are most commonly expressed as percents. So, when performing mathematical computations with them we need to express them as decimals.

Try this—Express each percent as a decimal and a fraction

12% = _____ 4% = _____

$\frac{1}{2}\%$ = _____ 4.5% = _____

Example: Find the annual simple interest rate if a principal of \$8000 yields \$8340 at the end of 1 year.

What is the amount of interest? _____ Let's use $I=Prt$

Try this--Steve opened a savings account that pays simple interest at the rate of $5\frac{1}{2}\%$ per year. If he deposits \$5000 and makes no other deposits, find the

- a. Interest and
- b. Amounts in the bank at the end of the following times:
 - a. 1 year
 - b. 3 months

FUTURE VALUE--If money (principal) is *invested* at $r\%$, then at the end of t years, the investor will have the *future value of the investment* – which is the principal *plus* the interest.

$$A = P + I$$

$$A = P + Prt$$

$$A = P(1 + rt) \quad \text{What do } P, r, t, \text{ and } A \text{ represent?}$$

Example: Find the:

- a. Annual simple interest rate if a principal of \$8000 yields \$8340 at the end of 1 year.
- b. The amount in the account after t years:

Let's use $A = P(1 + rt)$

Try These:

1. The future value of a three month investment of \$5000 is \$5300. What is the annual simple interest rate?

2. Suppose you borrow \$8,000 at 4% simple interest.
How much interest will you owe at:
 - a. the end of 1 year
 - b. the end of 9 months

- 2b. How much will you pay back at the end of each of the above time periods?

In contrast to *future value*, the principal that you have to invest in an account now to have a specified amount in the account in the future is called the *present value*.

PRESENT VALUE: SIMPLE INTEREST

If you want to earn an annual rate of 10% on your investment, how much (to the nearest cent) should you pay for a note that will be worth \$5,000 in 9 months.

Compound interest-pays interest on interest—It is where your money earns interest, but the interest is reinvested and then during the next payment period your money and interest earned in the previous period earns interest.

Example: If you invest \$100 at 8% interest compounded quarterly, how much will you have in the bank after 1 year?

Let's use the simple interest formula:

1st quarter $I = 100(0.08)(1/4) = 2$

$A = 100 + 2 = 102 = 100(1.02)$

2nd quarter $I = 102(0.08)(1/4) = 2.04$

$A = 102 + 2.04 = 104.04 = 100(1.02)(1.02) = 100(1.02)^2$

3rd quarter $I = 104.04(0.08)(1/4) = 2.08$

$A = 104.04 + 2.08 = 106.12 = 100(1.02)(1.02)(1.02) = 100(1.02)^3$

4th quarter $I = 106.12(0.08)(1/4) = 2.12$

$A = 106.12 + 2.12 = 108.24 = 100(1.02)(1.02)(1.02)(1.02) = 100(1.02)^4$

Period	Initial amount	Final amount
1	100	$100(1.02)$
2	$100(1.02)$	$100(1.02)(1.02) = 100(1.02)^2$
3	$100(1.02)^2(1.02)$	$100(1.02)^2(1.02) = 100(1.02)^3$
4	$100(1.02)^3$	$100(1.02)^3(1.02) = 100(1.02)^4$

Note this is a geometric sequence.

Notice that the interest earned each quarter keeps increasing. This is the effect of compound interest!

Each investment (or loan) has a defined number of payment periods:

annually - once per year

semiannually _____ per year

quarterly _____ per year

monthly _____ per year

daily _____ per year

In general, if the number of compounding periods increases, the result is an increase in the interest earned by the account.

FORMULA for computing the amount at the end of the n th period is

$$A = P (1 + r)^n$$

Note in this formula **r** is the interest rate **per period** and **n** is the **number of periods**.

Example: Suppose you deposit \$1500 in a savings account that pays 6% interest compounded monthly, what is the balance after 2 years?

$P =$ _____ $r =$ _____ $n =$ _____

Another Formula

AMOUNT (FUTURE VALUE) when interest is compounded:

$r =$ Rate per year (as a decimal)

$n =$ Number of compounding periods per
year

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$P =$ Principal or Present Value

$A =$ Amount Due or Future Value

$t =$ time in years

Let's use this formula to solve the same problem.

Example: Suppose you deposit \$1500 in a savings account that pays 6% interest compounded monthly, what is the balance after 2 years?

Always round to the nearest cent (unless told otherwise).

Try this--Find the value of \$1,000 invested at 8% for 5 years compounded:

- | | |
|--------------|-----------------|
| a. Annually | b. Semiannually |
| c. Quarterly | d. Monthly |

Notice how the value increases as the compounding periods increase.

Example: How much should you invest today at 10% compounded quarterly to have \$8,000 toward the purchase of a car 5 years from now? (Present Value)

EFFECTIVE ANNUAL YIELD

The **Effective Annual Yield** is the rate that a bank would have to offer if the principal will be invested using simple interest ($I = Prt$) instead of compound interest.

Example: Suppose that you invested \$1000 in a savings account that pays 6% interest compounded quarterly.

1. What is the balance at the end of one year?

2. What is the effective annual yield?

First we must find the amount of interest._____

Next we will use the simple interest formula_____ and solve for r.

Sometimes it is difficult to compare compounded interest rates.

Thus if you want to become an investor, it is difficult to make a wise decision as to which investment is the best.

Which has the higher return?

To compare you can use a set amount P (perhaps \$100) and set time period t (perhaps 1 year).

Bank	Rate	Compounded	Comparison
PNC	6.96%	daily (365)	<i>Lower rate, but compounded more</i>
IUS Credit Union	6.97%	quarterly	<i>Higher rate, but compounded less</i>

Which of the above is better?

Class work Interest Worksheet

Homework Course Compass Section 8.3 and page 509 #A

9, page 510 #B 2, 5, 7, 10, 12