Lesson 9.1(F'11)

Objectives: 1. To find the probabilities of events both mutually exclusive and not mutually exclusive.
2. To learn the properties of probability.

There are many applications of uses of probability in the sciences, in sports and in business. Probability is used in predicting sales, determining insurance premiums, testing experimental drugs, etc.

Examples: There is a 30% chance of rain.
What is the probability that IU will win?
What is the probability that a 40-year old will live to be 95?

Principles and Standards (page 516)
In grades K-2, children should discuss events related to their experiences as likely or unlikely.(p.400) etc.

Probabilities-are ratios that are expressed as fractions decimals or percents, determined by considering results or outcomes of experiments

TERMS

Experiment-is an activity where the results can be observed and recorded.

Outcome-each of the possible results of an experiment.

Example: What are the possible outcomes if we flip a coin? Use a tree diagram .

What are the possible outcomes if we roll a die? Use a tree diagram .

Experimental Probability- When a probability is determined by observing outcomes of experiments.

Flip a coin 20 times. How many heads did you obtain?___

The *relative frequency* is obtained by dividing the number of heads **actually observed** by the total number of tosses. Find the relative frequency for the number of heads observed in your 20 tosses

What is the relative frequency for everyone at your table?

MORE TERMS

Sample space-is a set of all possible outcomes for an experiment.

Event-is any subset of a sample space.

Example: Find the set of all **odd** numbered rolls of our die.

A = {1, 3, 5} is a subset of our **sample space**

S = {1, 2, 3, 4, 5,6 } and is called an **event**.

Example: If we have a jar with 10 different colored marbles (red, blue, yellow, pink, white, purple, black, orange, gray, green),
find each of the following:

1. The sample space---- S = {

2. The event A consisting of outcomes having a color beginning with the letter P.

   A = {

3. The event B consisting of outcomes having colors that have exactly 4 letters.

   B = {

4. The event C consisting of outcomes having colors that having 4 or 5 letters.

C ={

ACTIVITY Theoretical probability

Let's see if this is a fair game.

Let's list the sample space using ordered pairs (A,B).
(use a tree)

S = {(

How many cases will spinner A win?_____

How many cases will spinner B win?_____

Is it a fair game? ______

Law of Large Numbers (Bernoulli's Theorem)
If an experiment is repeated a large number of times, the experimental probability of a particular outcome approaches a fixed number as the number of repetitions increases.

Theoretical probabilities-outcomes under ideal conditions

Let's look at our marble example again. If we have a jar with 10 different colored marbles (red, blue, yellow, pink, white, purple, black, orange, gray, green),

P(B)- means the probability of getting a blue marble

P(B) = $\frac{1}{10}$ P(B) = P(Y) = P(G) = $\frac{1}{10}$

Equally likely-when one outcome is just as likely to occur as another.(fair)

If an experiment is repeated many times, the experimental probability of the event’s occurring should be approximately equal to the theoretical probability of the event occurring.

What is the theoretical probability of choosing a marble whose color has four letters(call this event F)?

S = { red, blue, yellow, pink, white, purple, black, orange, gray, green}

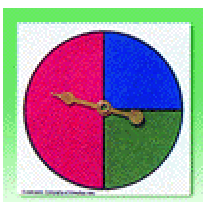
F = { blue, pink, gray }   P(F) = $\frac{3}{10}$

Definition of Probability of an event with equally likely outcomes-

For an experiment with **sample space** S and **equally likely** outcomes, the **probability** of an event A is given by

$$P(A) = \frac{Number\ of\ elements\ of\ A}{Number\ of\ elements\ of\ S} = \frac{n(A)}{n(S)}$$

NOTE- This is true only when the sample space has equally likely outcomes. If each possible outcome of the sample space is equally likely, the sample space is a **uniform sample space**.



Spinner not equally likely

Example: Let S = { 1, 2, 3, 4, …, 15}. If a number is chosen at **random**, that is equally likely, calculate each of the following probabilities.

1. The event A that a multiple of 5 is drawn.

A = {

$P(A) = \frac{n(A)}{n(S)} =$ ______

2. The event B that a number less than 10 is drawn.

B = {

$P(B) = \frac{n(B)}{n(S)} =$ ______

3. The event C that a number greater than 15 is drawn.

C = {

P(C) =_____

Impossible event

4. The event D that a number less than 16 is drawn.

D = {

PD) = _____

Certain event

5. The event E that a number greater than 15 or less than 5 is drawn.

E = {

P(E) = ____

Property

If A is any event and S is the sample space

$0\% \leq P(A) \leq 100\%$ or $0 \leq P(A) \leq 1$

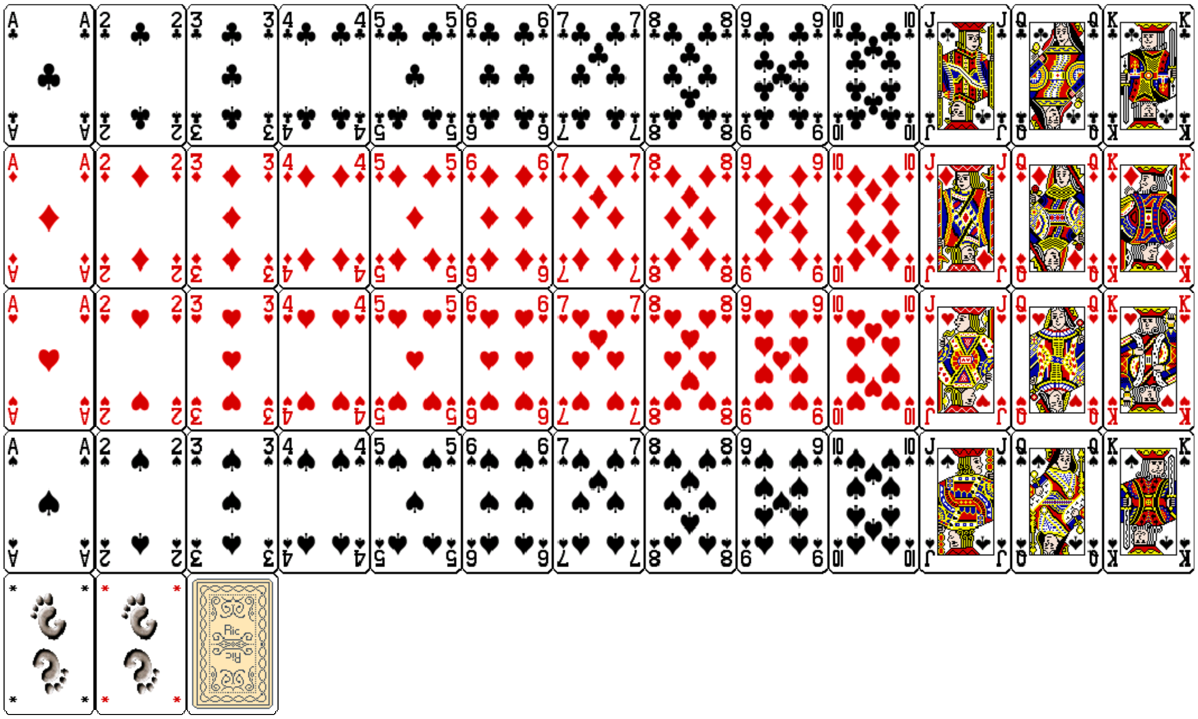
Consider event A above:

$P(A) = P(5) + P(10) + P(15) = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$

Property of Probability of an Event

The probability of an event is equal to the sum of the probabilities of the events representing all the outcomes in the original event.

Deck of cards



Example-If we pick a card at random from an ordinary deck, what is the probability that the card is a:

a. diamond (D).

b. diamond or a club P(B)

P(D) = $\dfrac{n(D)}{n(S)}$ = ______

P(B) = ______

Mutually Exclusive Events

Let event A be selecting a diamond and event B be selecting a club. $A \cap B = \varnothing$ **mutually exclusive events**

Definition of Mutually exclusive Events

Events A and B are mutually exclusive if $A \cap B = \varnothing$

The probability of A or B above (event A be selecting a diamond and event B be selecting a club)

P(A ∪ B) = $\frac{n(A \cup B)}{n(S)} = \frac{n(A)+n(B)}{n(S)} = \frac{13+13}{52} = \frac{1}{2}$

Property

If events A and B are mutually exclusive, then
P(A ∪B) = P(A) + P(B).

The probability of drawing a diamond is $\frac{13}{52}$.

What is the probability of not drawing a diamond?

(This is the **complemen**t of drawing a diamond)

$\frac{39}{52}$ P($\bar{A}$) = 1 – P(A)

Property

If A is a set and $\bar{A}$ is the compliment, then
P(A) + P($\bar{A}$) = 1 or P($\bar{A}$) = 1 – P(A) or P(A) = 1 – P($\bar{A}$)

Non-Mutually Exclusive Events

Consider event A of drawing a diamond and event B of drawing an ace.

n(A ∪ B) = _________ P (A ∪ B) = $\frac{n(A\cup B)}{n(S)} = \frac{\quad}{52} =$

Note: $n(A\cup B)$ = n(A) + n(B) – n($A\cap B$)

= 13 + 4 – 1 = 16

P(A ∪ B) =

$$\frac{n(A)+n(B)-n(A\cap B)}{n(S)} = \frac{13+4-1}{52} = \frac{16}{52} = \frac{4}{13}$$

PROPERTIES OF PROBABILITY page 441

1. P(∅) =_________ (Impossible event)

2. P(S) =_________, where S is the entire Sample Space (Certain event)

3. For any event A, _____ ≤ P(A) ≤ _____

4 A and B are events and A ∩ B = ∅, then P(A ∪ B) = _________________(Mutually Exclusive events)

5. If A and B are events, then P(A ∪ B) = ___________
(Non-Mutually Exclusive events.

6. If A is an event, then P($\overline{A}$) = ______________________
(Complementary Events)

Try this-- A bag contains 2 red marbles, 6 blue marbles and 3 yellow marbles.

1. What is the probability of event R that a red marble is drawn?

   R = {

   P(R) = ____

2. What is the probability of "not R" that a marble is drawn and it is not red?

   $P(\overline{R})$ = _____

3. What is the probability that a marble is either red or blue?

   $n(R \cup B)$=

   $P(R \cup B)$ = ______

Second Dice

First Dice	1	2	3	4	5	6
1	1,1 2	1,2 3	1,3 4	1,4 5	1,5 6	1,6 7
2	2,1 3	2,2 4	2,3 5	2,4 6	2,5 7	2,6 8
3	3,1 4	3,2 5	3,3 6	3,4 7	3,5 8	3,6 9
4	4,1 5	4,2 6	4,3 7	4,4 8	4,5 9	4,6 10
5	5,1 6	5,2 7	5,3 8	5,4 9	5,5 10	5,6 11
6	6,1 7	6,2 8	6,3 9	6,4 10	6,5 11	6,6 12

*The one is red is the sum

Example-Find the probability of rolling a sum of 5(A) or 6(B) when rolling a pair of die.

Let list the sample space.

Class work

Homework Course Compass Section 9.1 and page 528 #A 1, 2, 15, 16, page 530 #B 1, 2, 5, 10, 11