

Lesson 9.5_(F'11)

Objective: To use the fundamental counting principal to determine the number possible arrangements (permutations or combinations) of objects.

In this section we will learn how to count outcomes using 3 techniques:

1. Fundamental Counting Principle (A review from Chapter 2)
2. Permutations (where order matters! – that is being “first” is different than being “second”)
3. Combinations (where order is not important – just being a member the set is important)

THE FUNDAMENTAL COUNTING PRINCIPLE

Recall that we covered this in T 101, so let's try an example to count the number of ways a multi-stage experiment can turn out.

EXAMPLE: You have 3 pair of slacks and 2 shirts. How many different outfits can you make?
(Start with a tree diagram to count them.)

THE FUNDAMENTAL COUNTING PRINCIPLE:

If event M can occur in m ways and after it has occurred, event N can occur in n ways, then event M followed by event N can occur in _____ ways.

(Note: This principle generalizes to more than just 2 stage experiments.)

EXAMPLES:

1. How many meals are possible at a restaurant where you get to choose the salad, entree and dessert, if they have 4 types of salads, 5 different entrees, and 6 desserts?
2. If you are given a list of 5 TV shows and are asked to list your top 3 choices, how many ways could you do this?
3. How many different ID numbers are there if each is made up of 2 letters followed by 3 digits?

Suppose the letter and digits cannot be repeated?

Suppose the first letter must be I, for Indiana, and the letters and digits cannot be repeated?

4. In a horse race with 8 horses, how many different finishes (win, place, show) are possible?(THIS IS ALSO A PERMUTATION)

5. In how many ways can president, vice-president, secretary, and treasurer be chosen from a group of 10 people if no person can hold more than 1 office?
(THIS IS ALSO A PERMUTATION)

Permutation-is an arrangement of things in a definite order with no repetitions.

Name all the different arrangements of the numbers 1, 2, 3.

1 2 3 1 3 2 2 1 3 2 3 1
3 2 1 3 1 2

We can arrange the letters in 6 different ways.

We can use the Fundamental Counting Principal to determine the number of different arrangements:

$3 \times 2 \times 1 = 6$ We can write this as: ${}_3P_3 = 6$

In general if there n different objects then there are:
 $n(n - 1)(n - 2)(n - 3)(n - 4)\dots 1$ or $n!$ different arrangements

How many different ways can 8 different books be arranged on a shelf?

$${}_8P_8 =$$

Factorial 8!

What if we do not want to select only part of a group?

In how many ways can a contractor build different houses on 4 lots if there are six models to choose from?

This is a permutation of six objects taking four at a time.

Written as: ${}_6P_4$

In general if n objects are chosen r at a time, then the number of possible permutations, denoted by ${}_nP_r$, is

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$0! = 1$$

Try This--Find

1. ${}_7P_3$

3. ${}_5P_2$

2. ${}_8P_2$

4. ${}_9P_4$

5. How many win-place-show results are possible in a race with ten entries?
6. How many ways can you arrange 5 of ten dancers in a line?
7. How many ways can you arrange 3 of the letters from the alphabet if no letter can be repeated?

What if some of the object are repeated? Let's see.

How many ways can the letters SOS be arranged? Let's make a tree. Repeat for the letters BIG.

How many different ways were possible?

SOS _____ BIG _____ How are these different?

SOS SSO OSS 3 ways because there are 2 S's

$$\frac{{}_3P_3}{{}_2P_2} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6}{2} = 3$$

Permutation of Like Objects

If a set contains n elements, of which r_1 are of one kind, r_2 are of another kind, and so on through r_n , then the number of different arrangements of all n elements is equal to

$$\frac{n!}{r_1! \cdot r_2! \cdot r_3! \cdot \dots \cdot r_n!}$$

Find the number of arrangements of the letters in the word:

1. HAPPINESS

2. MISSISSIPPI

Combination

Combination-is an arrangement of objects in which the order is not important.

If a two person committee is selected with no chair from the students Sue, Tom, Don, how many different committees are possible?

First let's list all the possible permutations.

ST SD TS TD DS DT

Some of these are the same. Cross out all the same committees.

This leaves us with all the combinations.

ST SD TD Thus there are 3 committees possible

$\frac{3 \cdot 2}{2 \cdot 1}$ Which give the $\frac{\text{number of ways to choose 2 objects from 3}}{\text{number of repetitions}}$

$${}_3C_2 = \frac{3 \cdot 2}{2 \cdot 1} = \frac{6}{2} = 3$$

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

How many possible choices are there if you select to read three books from a list of 32?

Jamie normally invites the 14 girls in her class when she has a slumber party. She has been told to cut her guest list to 8. How many choices does she have as to whom to have?

Try this—All mixed up

1. How many meals are possible at a restaurant where you get to choose the salad, entrée and dessert, if they have 7 kinds of salads, 9 different entrees, and 5 different dessert choices?
2. How many ways could you arrange the letters in the word
 - a. PINK?
 - b. PEEP?
3. How many ways can you arrange 4 of 8 students in a line?
4. From a standard 52 card deck, how many different 5-card hands are possible?

A family decides to have five children. Since there are just two outcomes we will assume that each outcome is equally likely. What is the probability of having three boys?

A teacher needs to choose 3 students to run an errand out of a class of 35 students. How many ways could this be done?

The FDA is testing a new drug with 18 volunteers. Five will receive a placebo and the rest the drug. How many different placebo groups are possible?

How many different drug groups are possible?

At the beginning of the second quarter, each of a class's 25 students shook hands with each of the other students exactly once. How many handshakes took place?

How many different ways can the letters M,A,T,H be arranged?
What is the probability that it will spell MATH?

Class work

Homework Course Compass Section 9.5 and book page 584 #A
15, 17, 18, 19, 20 page 585 #b 1, 2, 3, 4, 8, 9, 10, 12, 13, 18