

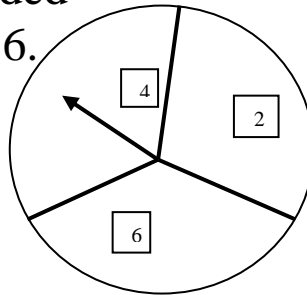
## Lesson 9.2 (F'11)

Objective: To determine the probability of multistage experiments.

A **TREE DIAGRAM** can be used to represent the outcomes of an experiment visually.

Let's make a tree diagram for a one stage event.

**EXAMPLE:** Spinning a spinner divided into three **equal** regions labeled 2, 4, 6.



Let's make a tree diagram for a two stage event.

Example-

A marble is drawn from a box and then **replaced** and another marble is drawn. If the marbles are **blue (B)**, **yellow (Y)** and **red(R)**, draw a tree diagram to list the combined outcomes of the two stage experiment.

Use the tree diagram to answer these questions.

1. Are the outcomes equally likely?
2. What is the sample space of this experiment?
3. What is the event of getting red both times?
4. What outcomes make up the event that both colors turn out the same?
5. What is the event of getting a red followed by a blue?

Now calculate the probabilities of each of the above events.

Example-A marble is drawn from a box and then **replaced** and another marble is drawn. If the marbles are **blue, blue**, and **red**, determine the probability of drawing RB

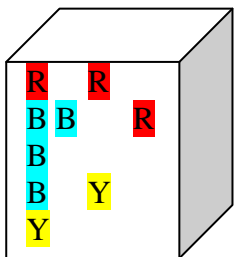
Property: Multiplication Rule for Probabilities for Tree Diagrams

For all multistage experiments, the probability of the outcome along any path of a tree diagram is equal to the **product** of all the probabilities along the path.

Example: A bead is drawn from a box and then **replaced** and another bead is drawn. If there are **3 red**, **4 blue** and **2 yellow** beads, what is the probability of selecting:

1. 2 red beads  $P(2 \text{ red}) = \underline{\hspace{2cm}}$
2. at least one red bead  $P(\text{at least one red}) = \underline{\hspace{2cm}}$
3. no red beads  $P(\text{no red}) = \underline{\hspace{2cm}}$

Let's draw a tree diagram.



What do you get if you add #2 and #3 above? Why?

## INDEPENDENT EVENTS

The color of the first ball has no effect on the color of the second ball because the first ball was replaced.

Two events are **independent** when the outcome of the first has no influence on the outcome of the second.

### PROPERTY

For any independent events A and B

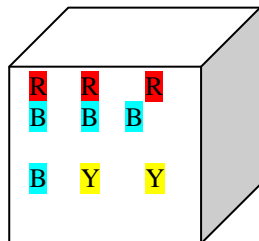
$$P(A \cap B) = P(A) \cdot P(B)$$

In the next example the events are not independent. (**dependent**)

Example: A bead is drawn from a box and **NOT Replaced** and another bead is drawn. If there are **3 red**, **4 blue** and **2 yellow** beads, what is the probability of selecting:

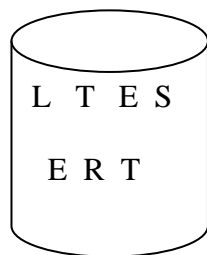
1. 2 red beads  $P(2 \text{ red}) = \underline{\hspace{2cm}}$
2. at least one red bead  $P(\text{at least one red}) = \underline{\hspace{2cm}}$
3. no red beads  $P(\text{no red beads}) = \underline{\hspace{2cm}}$

Let's draw a tree diagram.



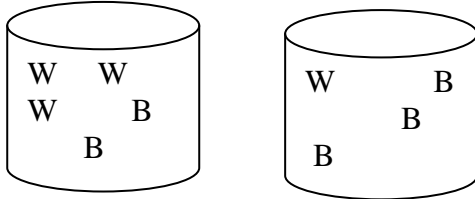
What do you get if you add #2 and #3 above?

Example: The letters in the word LETTERS are placed in a cylinder and drawn one by one without replacement. What is the probability of the outcome SEEL with the letters chosen in exactly that order.  
Do we need a complete tree diagram?



What if the letters are replaced?

**EXAMPLE:** You have 2 urns. Urn 1 contains 3 white and 2 black marbles. Urn 2 contains 1 white and 3 black marbles. The experiment is to draw a marble from Urn 1, note its color and place it in Urn 2. Then draw a marble from Urn 2 and note its color.



Are these events independent? \_\_\_\_\_

Tree Diagram:

Find:  $P(\text{both marbles are the same Color}) = \underline{\hspace{2cm}}$

$P(\text{the marble drawn from Urn 2 is white}) = \underline{\hspace{2cm}}$

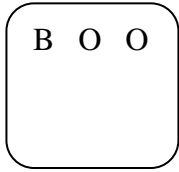
$P(\text{at least one marble drawn is white}) = \underline{\hspace{2cm}}$

$P(\text{neither of the marbles is white}) = \underline{\hspace{2cm}}$

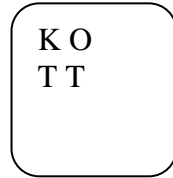
**NOTE THAT THE LAST TWO QUESTIONS ARE COMPLEMENTS OF EACH OTHER. DO THEIR PROBABILITIES SUM TO 1?**

Try This--There are two boxes. Box 1 contains the letters B O O, and Box 2 contains the letters K O T T. The experiment is to draw a letter from box 1 and place it in box 2. Then you draw a letter from box 2 and note its name. What is the probability that the letter selected from box 2 will be an O?

BOX 1



BOX 2



Could we have use just the letters O and  $\bar{O}$  in our tree diagram?

Let's try it together.

**EXAMPLE:** A committee consists of 10 members: 4 women and 6 men. Three members are selected at random to be sent to a meeting in Hawaii. The three names were drawn out a hat and all three were women's names. What is the probability of such luck? Do we need a complete tree? (Is this independent?)

Brittany is going to ascend a four step staircase. At any time, she is just as likely to stride up one step or two steps. Find the probability she will ascend the four steps in:

1. 2 strides
2. 3 strides
3. 4 strides

**EXAMPLE:** An assembly line has two inspectors. The probability that the first misses a defective item is .05. If the defective item passes the first inspector, the probability that the second inspector will miss it is .01. (Is this independent?)

- a. What is the probability that a defective item will pass by both inspectors?
- b. What is the probability that a defective item will not pass by both inspectors?

### **PROBLEMS INVOLVING THE PHRASE “AT LEAST ONE”**

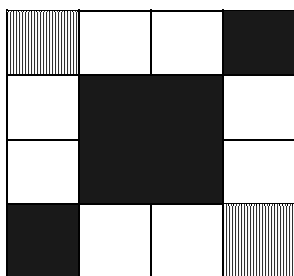
In these cases it is often more useful to take advantage of the complement. Since “at least one” is the complement of “none”, we can say:  $P(\text{at least one}) = 1 - P(\text{none})$

**EXAMPLE:** A husband and wife discover that there is a 10% probability of passing on a hereditary disease to each of their children. If they plan to have 3 children, what is the probability that at least one child will inherit the disease? (Is this independent?)

## GEOMETRIC PROBABILITY

Geometric probability (also known as an AREA MODEL) uses geometric shapes to help determine probabilities. The geometric region represents the sample space and the shaded area(s) the event(s) in question. Consider the following dart board.

What is the probability of hitting any non-white region?  
What is the probability of hitting a striped region?



Try this--Given the following dart board, what is the probability of hitting each section?

B	B	B
B	A	B
C	C	C

$$P(A)=\underline{\hspace{2cm}} \quad P(B)=\underline{\hspace{2cm}} \quad P(C) =\underline{\hspace{2cm}}$$

Class work

Course Compass section 9.2 and book #549 #A 4, 8, 10, 11  
551 #B 2, 3, 5, 7, 10, 16