

## Lesson 6.3 F'09

Objectives: 1. To convert fractions to decimals that repeat and repeating decimals to fractions.

Any rational number can be expressed as a decimal that must either terminating or repeating.

Remember from Section 6.1

Given the rational number  $\frac{a}{b}$  in simplest form (reduced): If the prime factorization of  $b$  contains only \_\_\_\_\_'s and/or \_\_\_\_\_'s then  $\frac{a}{b}$  can be written as a \_\_\_\_\_ decimal.

If the prime factorization of  $b$  contains factors other than \_\_\_\_\_'s and/or \_\_\_\_\_'s then  $\frac{a}{b}$  can be written as a \_\_\_\_\_ decimal.

Consider  $\frac{9}{40}$  Does it terminate? \_\_\_\_\_

Let's change it to a decimal. Consider two different ways.

Method 1: Make denominator a power of 10

Method 2: Division

## Repeating decimals

Find the decimal for each fraction.

1.  $\frac{3}{7}$

2.  $\frac{4}{9}$

3.  $\frac{5}{11}$

4.  $\frac{22}{45}$

The above problems are examples of a *repeating decimals*, and the repeating block of digits is called the *repetend*. We use a bar to indicate that the block of digits underneath is repeated continuously.

EXAMPLES: Change each of the following to a decimal without using a calculator

$$\frac{5}{6}$$

$$\frac{3}{8}$$

$$\frac{5}{11}$$

$$\frac{1}{7}$$

As we went through the division process in the last example  $\frac{1}{7}$ , we obtained remainders of 3, 2, 6, 4, 5, and 1. These are *all the* possible nonzero remainders when dividing by 7– that is 6 possible remainders. (If we got a 0, it would have terminated.). Since each of the possible remainders had occurred, at this point, one of the possible remainders *must* reoccur in the next subtraction. This is where the repetend for  $\frac{1}{7}$  must begin again (if not before) and it did! In general, if  $\frac{a}{b}$  is any rational number in simplest form and does not terminate, the repetend will have at most \_\_\_\_\_ digits.

Now, let's try one a bit more challenging. Using the above information, how many digits are possible in the repetend of  $\frac{1}{17}$ ?

\_\_\_\_\_

Now, let's divide – using our calculator and a new method to help us since we know the repetend could possibly be quite lengthy.

$$\frac{1}{17} =$$

Try This:  $\frac{1}{19}$

## Repeating decimal as a fraction

We know that  $0.48 = \quad =$

But when we have a repeating decimal we have infinitely many digits, so there is no single power of 10 that can be placed in the denominator. To overcome this difficulty, we must somehow eliminate the *infinitely repeating* part of the decimal by multiplying by a power of 10 *and use a little bit of algebra*.

1.  $0.\overline{6}$

2.  $0.\overline{72}$

2.  $0.\overline{876}$

4.  $3.\overline{245}$



## **DENSENESS PROPERTY OF DECIMALS**

Between any two rational numbers there are infinitely many rational numbers.

Find three rational numbers between  $0.\overline{69}$  and  $0.\overline{691}$

Activity WORKSHEET

Homework Math XI section 6.3 and book #1, 2, 7, 13, 15,  
17