Lesson 5.5 and 5.7 Fall 2009

Objectives: To find the future value of investments that are compounded:

a. annually
b. k times a year
c. continuously

To find the present value of an investment
To work with logistic functions.

<u>**Future Value of an Account**</u>

<u>Terminology:</u>

P: The Principal (the original amount of the loan or account)

S: The Future Value (the principal plus all interest earned or charged)

r: the annual rate of interest converted from a percent to a decimal. Also called nominal interest rate, or the rate

t: the number of <u>years</u> involving the loan or account

k: the number of times per year that interest is compounded. k varies – common values are in the table below.

If interest is compounded:	Then k is:
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

From Section 3.4, we found that if a set of data has an initial value ***a*** and a constant percent change r for equally spaced inputs, x, the data can be modeled by the exponential function: $\mathbf{y = a(1 + r)^x}$

We can use this formula to find the future value of an account.
This formula can lead us to the following:

FUTURE VALUE OF AN INVESTMENT
WITH ANNUAL COMPOUNDING

If \$P is invested at an interest rate r per year, compounded annually, the future value S at the end of t years is :

$$S = P(1+r)^t$$

FUTURE VALUE OF AN INVESTMENT
WITH PERIODIC COMPOUNDING

If \$P is invested for t years at an interest rate r, where the interest is compounded k times per year, then the interest rate per period is $\frac{r}{k}$, the number of compounding periods is kt, and the future value that results is given by:

$$S = P\left(1+\frac{r}{k}\right)^{kt} \text{ dollars.}$$

EXAMPLE: #16 Suppose $6400 is invested for *x* years at 7% interest compounded annually. Find the future value of this investment at the end of

a. 10 years   b. 30 years

Let's use our new variables: $S = 6400(1 + .07)^t$.

Now substitute in 10 for x and then 30 for x.

EXAMPLE #17 If $3300 is invested for x years at 10% interest compounded annually, the future value that results is $S = 3300(1.10)^x$ dollars.

a. Graph the function for x = 0 to x = 8.

b. Use the graph to estimate when the money in the account will double.

EXAMPLE: Consider $1000 invested for 4 years at 6%, compounded:

a) Annually   b) Semiannually

c) Quarterly   d) Monthly

e) daily   f) hourly

The future value of an account that is compounded x times per year becomes

$f(x) = A(1 + \frac{1}{x})^x$

From the above example, the future value of our account increases as the number of compounding periods during the year increases. As x gets very large, the future value approaches the number e, which is 2.718281828 to nine decimal places. If our inputs get larger and larger the output that results approach the number e.

Future Value of an Investment with Continuous Compounding

$$S = Pe^{rt}$$

Find the future value of $1000 at 6% for 4 years if compounded continuously.

EXAMPLE (page 340)#30 Find how long it takes for an investment to double if it is invested at 6% interest. Solve graphically and analytically.

a. compounded annually b. Compounded continuously

EXAMPLE (page 340#32 If $12,000 is invested in an account that pays 8% interest compounded quarterly, find the future value of this investment

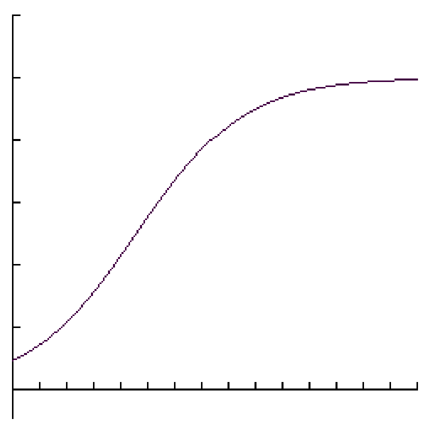
a. After 2 quarters. b. After 10 years.

3.7 Logistic Functions

Logistic Function

For real numbers *a, b,* and *c,* the function $f(x)=\frac{c}{1+ae^{-bx}}$ is a logistic function. If $a > 0$, a logistic function increases when $b > 0$ and decreases when $b < 0$.

Logistic Growth Function

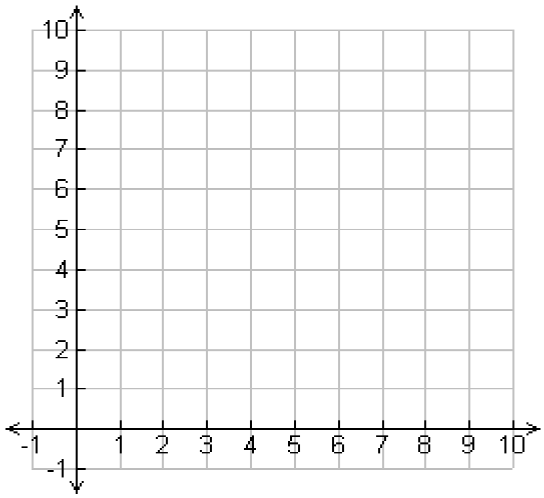


Equation	$y=\frac{c}{1+ae^{-bx}} \quad (a>0, b>0)$
***x*-Intercept**	**none**
***y*-intercept**	**(0,** $\frac{c}{1+a}$ **)**
Horizontal Asymptote	**:** ***y = 0, y = c***

The number, *c*, is called the *limiting value* or the *upper limit* of the function because the graph of a logistic growth function will have a horizontal asymptote at *y* = *c*.

EXAMPLE (page 356) #12 The percent of girls between ages 15 and 20 that have been sexually active at some time (the cumulative percent) can be modeled by the logistics function $y = \frac{83.84}{1+13.9233e^{-0.9248x}}$ where x is the number of years after age 15

a. Graph the function from $0 \le x \le 5$.



b. What does the model estimate the cumulative percent to be for girls of age 16?

c. What cumulative percent does the model estimate for girls of age 20?

d. What is the upper limit implied by the given logistic model?

EXAMPLE(page 357): #16 The following table gives the population of Japan for the year 1984-1999.

Year	Population (millions)	Year	Population (millions)
1984	120.235	1992	124.452
1985	121.049	1993	124.764
1986	121.672	1994	125.034
1987	122.264	1995	125.570
1988	122.783	1996	125.864
1989	123.255	1997	126.166
1990	123.611	1998	126.486
1991	124.043	1999	126.686

a. Find the logistic function that models the population *N*, using an input *x* equal to the number of years from 1980

b. Comment on the goodness of fit of the model to the data

Homework Course Compass sections 5.5 and 5.7 and page 387 #15, 19, 21, 25-33 odd page355 1, 5, 9, 11, 13, 15, 25