Lesson 5.2

Objectives: To convert an equation from exponential form to logarithmic form.
To determine the base of a logarithm.
To find common and natural logarithms.
To solve logarithmic equations.
To know and use the properties of logarithms

A logarithm function is the inverse of an exponential function.

Let $y = b^x$, $b > 1$ and $x > 0$, Then the inverse of this function is $x = b^y$. In order to solve this function for y we need a new notation. That notation is called a logarithmic function.

Logarithmic Function

For $x > 0$, $b > 0$, *and* $b \neq 1$, the logarithmic function to the base *b* is $y = \log_b x$ which is defined by $x = b^y$. This function is the inverse function of the exponential function $y = b^x$.

EXAMPLE: Write each of the following in exponential form.

1. $x = \log_4 64$ 2. $x = \log_3 27$ 3. $x = \log_5 125$

YOU TRY:

1. $y = \log_3 9$ 2. $2 = \log_7 y$ 3. $y = \log_4 64$

4. $y = \log_2 z$ 5. $y = \log_2 8$ 6. $y = \log_6 t$

EXAMPLE: Write each of the following in logarithmic form.

1. $y = 3^x$ 2. $y = 4^z$ 3. $y = 7^x$

YOU TRY:

1. $y = 3^x$ 2. $7 = 3^a$ 3. $27 = a^3$

EVALUATE:

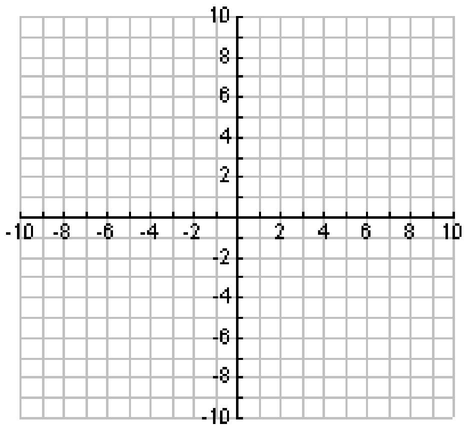
1. $\log_3 27$ 2. $\log_4 64$ 3. $\log_5 25$

4. $\log_9 3$ 5. $\log_3 \frac{1}{9}$ 6. $\log_9 27$

Graphing a logarithmic function

Example: Graph $y = \log_2 x$ by hand.

First let's write the equation in exponential form. $2^y = x$.
Now let's make a table choosing values for y and solving for x. Then graph these values.



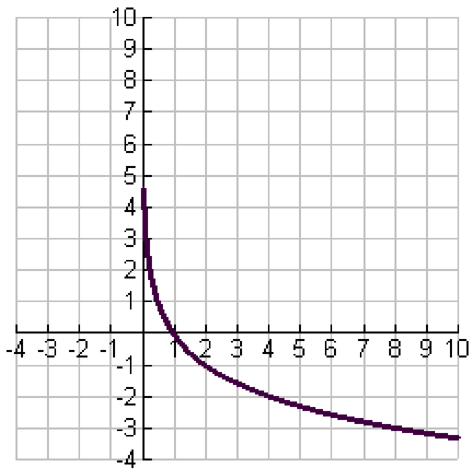
x	y
	-3
	-2
	-1
	0
	1
	2
	3

Does this graph have an asymptote? What is it?__________

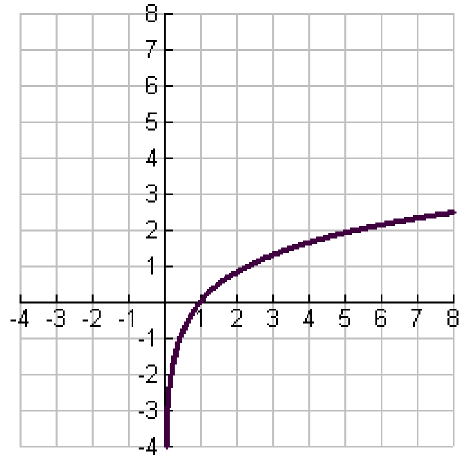
Properties of Logarithmic Functions

Equation: $y = \log_b x \quad (b > 0, b \neq 1)$

x-intercept: (1,0)
y-intercept: none
Domain: $x > 0$
Range: all real numbers
Vertical asymptote: *y*-axis (the line $x = 0$)



y = log $_b$ x (0 < b < 1)



y = log $_b$ x b > 1

Common Logarithm

If the base of our logarithm is 10 or *e*, we can use our calculator to evaluate the logarithm. $\log_{10} x = \log x$

EXAMPLE: Use your calculator to find:
1. log 100   2. log (0.01)   3. log 1000   4. log (0.1)

Richter Scale

If *I* is the intensity of an earthquake and I_o is a certain minimum intensity used for comparison, then the magnitude R of an earthquake of intensity *I* is:

$$R = \log\left(\frac{I}{I_o}\right)$$

This essentially “scales down” the measurements.

EXAMPLE:)#54 a) If an earthquake has an intensity of 250,000 times I_o, what is the magnitude of the earthquake?

b) Compare the magnitudes of two earthquakes when the intensity of one is 10 times the intensity of the other.

Richter Scale

1. If the intensity of an earthquake is I, its Richter scale measurement is $R = \log\left(\frac{I}{I_o}\right)$.

2. If the Richter scale reading of an earthquake is k, the intensity of the earthquake is I = $10^k I_0$.

3. If the difference of the Richter scale measurements of two earthquakes is the positive number *d*, the intensity of the larger earthquake is 10^d times more than that of the smaller earthquake.

Natural Logarithm

The logarithmic function with base e is called a natural logarithm. ($y = \log_e x = \ln x$)

EXAMPLE: Use your calculator to find each of the following to four decimal places.

1. ln 3.5 2. ln 0.45 3. ln 1.56

Doubling Time

If P dollars are invested for t years at an annual interest rate r compounded continuously, then the investment will grow to a future value S given by the exponential function $S = Pe^{rt}$ and the investment will be doubled when S = 2P, giving $2P = Pe^{rt}$ or $2 = e^{rt}$. If we change this equation to its logarithmic form we have $\log_e 2 = rt$ or ln 2 = rt. Solving this equation for t gives the doubling time.

Therefore **t =** $\frac{\ln 2}{r}$ is the time it take for an investment to double.

EXAMPLE: #48 If $5400 is invested in an account earning 7% annual interest compounded continuously, then the number of years that it takes for the amount to grow to $10,800 is $n = \frac{\ln 2}{0.07}$. Find the number of years.

EXAMPLE: #45For the years 1970-2006, the percent of females in the workforce is given by y = 11.101 + 8.090ln x, where x is the number of years from 1960.

a. What does the model predict the percent to be in 2011? In 2015?

b. Is the percent of females increasing or decreasing?

Property	Exponential Form
1. $\log_b b = 1$	1. $b^1 = b$
2. $\log_b 1 = 0$	2. $b^0 = 1$
3. $\log_b b^x = x$	3. $b^x = b^x$
4. $b^{\log_b x} = x$	4. $\log_\text{b} x = \log_\text{b} x$

Examples: Use the basic properties of logarithms to simplify.

1. $\log_3 3^4$ 2. $\log_7 7$ 3. $\log_5 1$

4. $\log 10^4$ 5. $\ln e^3$ 6. $\log\left(\frac{1}{1000}\right)$

Additional Logarithmic Properties (or "log laws")

For $b > 0, b \neq 1$:

5. **Product Property** $\log_b MN = \log_b M + \log_b N$

6. Quotient Property $\log_b \frac{M}{N} = \log_b M - \log_b N$

7. Power Property $\log_b M^k = k \log_b \mathbf{M}$

EXAMPLES: Rewrite the following expressions as the sum, difference, or product of logarithms, and simplify if possible.

1. $\log_4 3(x - 2)$

2. $\ln(e^3(e - 2))$

3. $\log \left(\frac{x-2}{x}\right)$

4. $\log(x^3(x - 2)^4)$

EXAMPLE: Rewrite the following expressions as a single logarithm.

a. $\log x + 3 \log y$

b. $2 \log_3 a - 3 \log_3 b$

c. $\ln (4x) - 4 \ln x$

EXAMPLE: Use logarithmic properties to find each of the following given that $\log_x 5 = a$ and $\log_x 6 = b$.

a. $\log_x 30$

b. $\log_x 25$

c. $\log_x 180$

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