

Lesson 5.1_(F'09)

- Objectives: To review the laws of exponents.
To graph exponential functions, finding the horizontal asymptotes.
To work exponential growth and decay problems.
To introduce the number e .
To transform the graphs of exponential functions.

Laws of Exponents

If m , n , a and b are real numbers $a > 0$ and $b > 0$, then

$$1. b^n \cdot b^m = b^{n+m}$$

$$5. \frac{b^n}{b^m} = b^{n-m}$$

$$2. (b^n)^m = b^{nm}$$

$$6. 1^m = 1$$

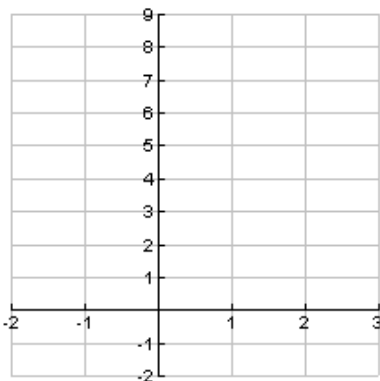
$$3. (ab)^n = a^n \cdot b^n$$

$$7. \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$4. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$8. b^0 = 1$$

EXAMPLE: Graph $y = 3^x$. State the domain and range.
Let's begin by making a table. Note the horizontal asymptote.



x	y
-2	
-1	
0	
1	
2	

Asymptote _____

Note: The domain is all real numbers, the range is $y > 0$.
The horizontal asymptote is the x axis.

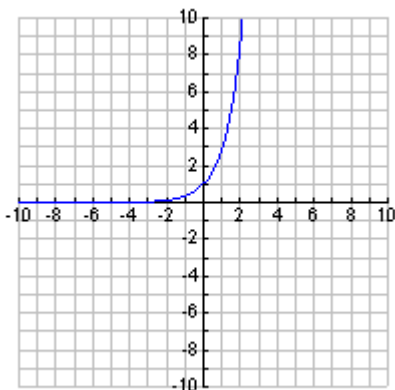
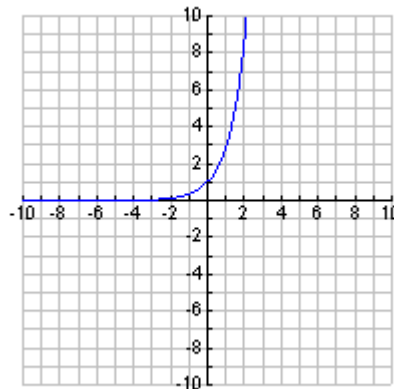
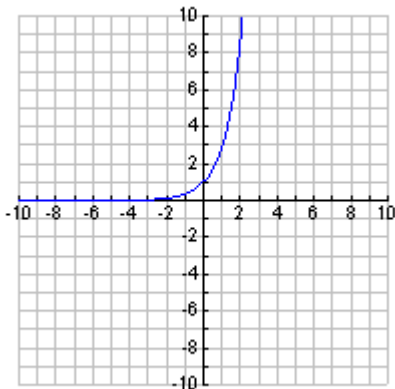
Exponential Function

If b is a positive real number, $b \neq 1$, then the function $f(x) = b^x$ is an exponential function. The constant b is called the *base* of the function and the variable x is the *exponent*.

Transformations of Graphs of Exponential Functions

Graph $y = 3^x$ 1. $y = 3^{-x}$ 2. $y = 3^{x+1} - 2$ 3. $y = -3^x$

Note $y = 3^x$ is already graphed on each graph.



EXAMPLE: Explain how the graph of :

1. $y = 3^{x+4}$ 2. $y = 3^x - 2$ 3. $y = 3^{x+4} + 2$

compare to $y = 3^x$

Exponential Growth Function

Equation: $y = a(b^x)$ $b > 1, a > 0$

x-intercept: none

y-intercept: $(0, a)$

Domain: All real numbers

Range: All real numbers $y > 0$

Horizontal asymptote: x – axis (the line $y = 0$)

Shape: Increasing on domain and concave up

EXAMPLE: #42 An antique table increases in value according to the function $v(x) = 850(1.04^x)$ dollars, where x is the number of years after 1990.

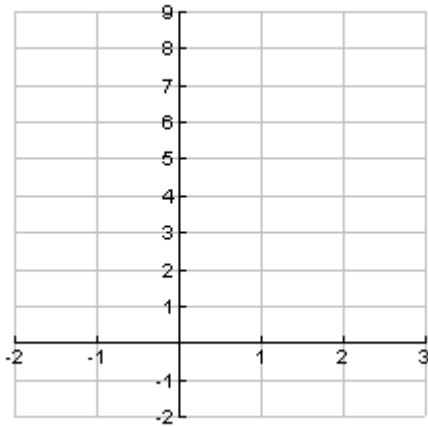
- a. How much was the table worth in 1990?

- b. If the pattern indicated by the function remains valid, what was the value of the table in 2005?

- c. Use a table or graph to estimate the year when this table will reach double its 1990 value.

Exponential Decay

Graph $y = \left(\frac{1}{3}\right)^x = 3^{-x}$



x	y
-2	
-1	
0	
1	
2	

Exponential Decay Function

Equation: $y = a(b^{-x})$, $b > 1$, $a > 0$

x-intercept: none

y-intercept: $(0, a)$

Domain: All real numbers

Range: All real numbers $y > 0$

Horizontal asymptote: x-axis (the line $y = 0$)

Shape: Decreasing on domain and concave up

EXAMPLE: #44 The population in a certain city was 800,000 in 2003, and its future size is predicted to be $P = 800,000e^{-0.020t}$ where t is the number of years after 2003.

- a. Does this model indicate that the population is increasing or decreasing?

- b. Use this model to predict the population of the city in 2010.

- c. Use this model to predict the population of the city in 2020.

- d. What is the average rate of change in population between 2010 and 2020?

Growth and Decay

For initial amount $y_0 > 0$, the equation $y = y_0 b^{kx}$ $b > 1$ defines a growth function if $k > 0$ and a decay function if $k < 0$.

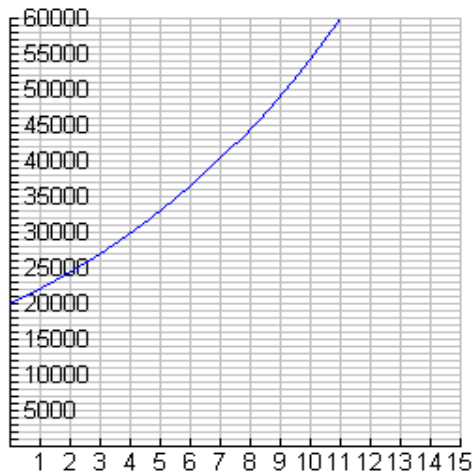
The Number e

The number e is an irrational number with decimal approximations 2.718281828 (to nine decimal places). It occurs frequently in business and science. The graph is near the graph of above the graph of $y = 2^x$ and below the graph of $y = 3^x$.

Growth of an Investment

If \$20,000 is invested for 12 years at 10%, compounded continuously, the future value is given by $S = 20,000e^{0.10t}$ dollars.

a. Graph this function for $0 \leq t \leq 7$.



b. Use this graph to estimate when the future value will be \$50,000.

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